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A STUDY OF UNCERTAINTY EVALUATION IN TRANSMISSION MEASUREMENT IN GAMMA RAY TOMOGRAPHY

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Abstract: A study of uncertainty in gamma ray transmission measurement of single beam tomography is presented. The radial density distribution of the FCC (fluidized catalytic cracking) catalyst in experimental riser is shown in measured and calculated data. Discrete models are proposed to describe data and for a better understanding of density distribution measurements. Validation procedure for the measurement models and uncertainty evaluation were carried out.

Keywords: discrete models, standard uncertainty, single beam tomography.

1. INTRODUCTION

The pioneer work on gamma ray tomography, the R.N. Bartholomew and R.M. Casagrande (1957) paper, by measuring solid concentrations in FCC riser, gives an expression to estimate experimental errors. In this work, for calculating the solid density average in the gamma ray path length, the following equation is proposed

$$\rho = \frac{1}{\alpha D_i} \ln \frac{I_V}{I_F} \qquad (1)$$

where α is the mass attenuation coefficient, D_i is internal diameter, and I_V , I_F are the gamma ray intensities in empty and at flow riser conditions. A much higher contribution from the gamma ray intensity measurement is reported in the mentioned [1] paper, according to the errors evaluation. The statistical uncertainty is calculated in [2], by means of the error propagation formula where only intensity I and gamma ray counts N observed, are estimated. Special attention is devoted to errors increase in chord length along tube radius [2], nearing the riser walls. By using the Equation (1) or another Beer-Lambert based equation, that relates the linear attenuation coefficient μ to the intensity

$$\frac{I}{I_0} = \exp\left[-\int \mu ds\right] \qquad (2)$$

where s is the gamma ray path length, to scan the riser, the internal diameter D_i in Equation (1), should be replaced by the chordal length c_i inside riser, measured at a number of i gamma ray trajectories. Applying GDT- gamma densitometry tomography to industrial scale bubble columns, a uncertainty analysis is carried out taken into account several sources of uncertainty as instrumental errors, including Compton scattering effects, statistical uncertainty in gamma counts, in flow variation and the image reconstruction errors [3]. The length L inside riser,

$$L = \frac{-1}{\mu} \ln \frac{I}{I_0} \qquad (3)$$

is calculated and the known water length is taken as a reference for the measurements. Some expressions are derived for the uncertainty evaluation and an increase in the error near the walls measurement cause large inaccuracy on the reconstruction data at this location. The chosen function form that relates intensity to the attenuation coefficient the Equation (2), is thought also as a possible source of error [3]. Additional reports that describe wall effects influencing sensing techniques in X-Ray tomography can be found [4].

Tomographic parameters as density resolution, spatial resolution and temporal resolution are usual indicators of the imaging process capability. Temporal resolution is probably the most questionable parameter in gamma ray tomography, while competition among other techniques is thought of [5]. To follow the competition the gamma ray tomography developed a fun beam with small scintillation detectors plus the gamma source to be packed into the arrangement geometry [6]. For single beam tomography the precise definition of parameters and the uncertainty evaluation of the measurements [7], might be a way to access new industrial contribution, on the field. The mathematical reconstruction works on the data matrix from transmission measurements along the riser radius. A relationship follows from direct measurements to the inverse model [8]. That a small uncertainty in data can lead to a large uncertainty in the result is demonstrated. The paper suggests that, a limit to the inversion procedure and experimental evaluation of the metrological performance of the algorithm should be carried out.

2. PARAMETER ESTIMATIONS

Several measurement uncertainty are involved in the transmission measurement of the riser, and a possible source of uncertainty is the form of Equation (2), as it is pointed out by [3]. Such a possibility was investigated once in the original Equation (2), the I_0, I intensities changed to I_V, I_F as given in Equation (1), in order to match riser measuring conditions. In this conditions, Equation (3) also was adapted to the I_V, I_F intensities. The meaning of calculating ρ with Equation (1), or L with Equation (3), inside riser, might be better observed in Figure 1, in single beam geometry

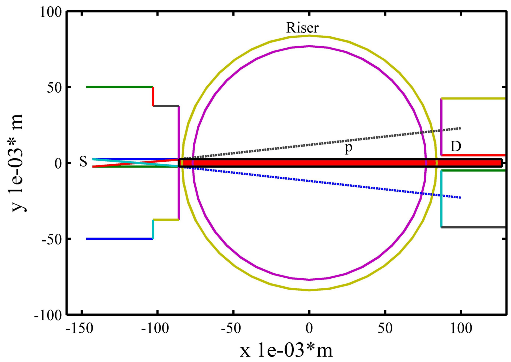

Fig. 1. A view of the gamma ray transmission geometry with S source, beam penetrating Riser, penumbral region p, and Detector collimator aperture.

In Figure 1, the gamma ray path length cross the tube walls to be detected, in while, in the givens equations the intensities are evaluated inside riser. The transmission measurement by scanning is taken along x-axis in a coordinate system where the origin coincides with the geometrical riser center, Figure (1). As the chordal length decreases and the tube thickness increases nearing the wall the gamma ray attenuation increases. The obtained gamma profile has such a parabolic curve form that data could not be well fitted by any given model in [9], for example. Such a behavior requires a model that takes into account the geometry of the tube. Then tube thickness d was modeled as a function d = f (Re, R) of the external and internal radii that can be observed in Figure (1). To the I_V intensity calculation was proposed [10] the equation

$$y_V = a_1 \exp(a_2 d) \qquad (4)$$

where linear and nonlinear parameters are fitted to data by least square method. The I_F is also obtained by the expression

$$y_F = a_1 \exp(a_2 d - \mu c_o) \qquad (5)$$

where c_o is the chordal length of the scanned object. Now both intensities are calculated to predict the radial density distribution. Replacing these intensities in Equation (1)

$$\rho_m = \frac{1}{\alpha c_i} \ln \frac{y_V}{y_F} \qquad (6)$$

with c_i the chordal length inside riser, for measurements taken along radius at i points. Equation (6) can be simplified to

$$\rho_m = \rho_c \frac{c_o}{c_i} \qquad (7)$$

where ρ_c is the catalyst apparent density. The terms on the right side of Equation (7), are known for a static experiment with catalyst, to calculate density distribution. The measurements by scanning the riser can be evaluated by comparing with values given by Equation (7), to analyze data prior to mathematical reconstruction and to be used a reference.

To model the spatial resolution the gamma beam width was measured inside riser. A two terms Gaussian function fitted the data in a sigmoid shape curve, then, by numerical differentiation a Gaussian peak was obtained. Using this peak curve a spatial resolution definition is proposed: the distance between peak centers $S \geq FWHM \cdot \sqrt{2}$. The FWHM – Full Width at Half Maximum is adopted as an already given criterion in gamma spectrum analysis [11]. In Figure 1 two peaks are placed on the same graph in order to illustrate the definition criterion

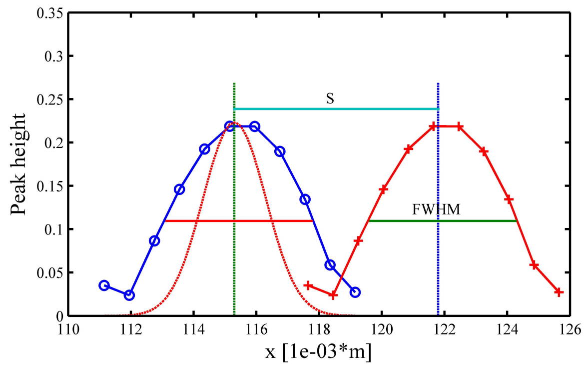

Fig. 2. Spatial resolution determination by means of the distance between peak centers S = FWHM.$\sqrt{2}$.

In Figure 2, circles are experimental data, the FWHM is indicated by horizontal line and dashed line is the Gaussian probability density function, to estimate the departure from symmetry.

By measuring three different gamma beam widths inside riser the spatial resolution definition was evaluated [12], as the minimum distance that two objects can be distinguished. The data matrix from the scan measurement should follow a sampling procedure, according to the Nyquist criterion, in order to avoid aliasing in imaging process. To predict catalyst flow measurement at different sampling frequency the gamma beam in a single interaction was modeled by a sum of sins function. In the multiple interactions, as in the catalyst static flow experiment a Fourier series fits the data well:

$$f(x) = a_0 + a_1\cos(xw) + b_1\sin(xw) + \cdots + a_8\cos(8xw) + b_8\sin(8xw) \quad (8)$$

where a and b, are fitted to data parameters and w is a frequency factor.

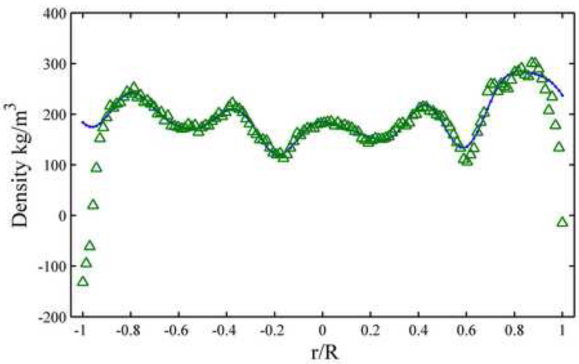

Fig.3. Radial catalyst density distribution in riser by gamma ray transmission measurement.

In Figure 3, experimental and calculated data are given in triangles and points. The measurement is quite well simulated the sampling interval r is in a r/R ratio to internal radius R of 0. 95, so in this interval, experimental and calculated data agrees in Figure 3. The experiment shown in Figure 4, the density distribution was measured in three different sampling rates

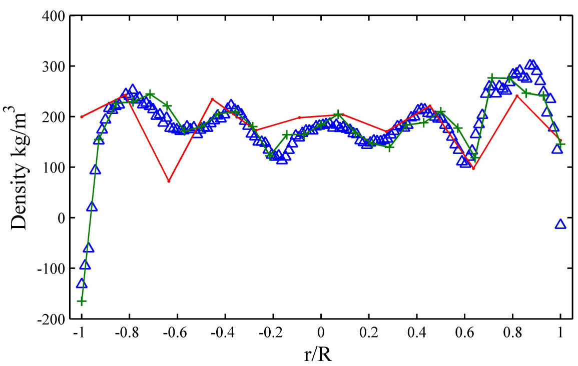

Fig. 4. Radial catalyst density distribution in riser measured at three different sampling rates.

In Figure 4, the gamma beam trajectories follow the sampling rates of 1 sample per 1.10^{-3} m in triangles, 1 sample per 5.10^{-3}m in crosses and 1 sample per 12.10^{-3}m is given in line. As expected, the highest sampling frequencies give the most accurate information about the radial distribution. According to Nyquist the highest one of 1 sample per 1.10^{-3} m is over sampled and the others two sampling rates are under sampled. Trying to optimize by seen, the second sampling rate of 1 sample per 5.10^{-3}m, given in crosses, might be acceptable for a radial distribution investigation, but it probably will cause aliasing, later at the reconstruction imaging process. At the lowest sampling rate, surely, the data points are missing the trend of catalyst flow.

Mathematical reconstruction. By using a triangular Bézier patch was proposed, based on a fixed 3-projection approach [13], for which a trajectory is easily described in barycentric coordinates. To reconstruct the density function through a functional tensor product Bézier surface [14], based on an arbitrary number of projections, is now under investigation. This function is a parameterized barycentric combination of uniformly spaced points, called control points, or control densities:

$$b(s,t) = \sum_{i=0}^{n}\sum_{j=0}^{m} B_i^n(s) B_j^m(t) b_{ij} \quad (9)$$

where $b(s,t)$ is the function value, s and t are the parameters with values between 0 and 1, b_{ij} are the control densities, $B_k^p(r) = \binom{p}{k}(1-r)^{p-k} r^k$ are the Bernstein polynomials, and p is their degree. A gamma ray trajectory is a line given by an equation in terms of s and t: $s=at+b$. The graph of $b(s,t)$ interpolates the points b_{ij} where $i=0$ or $i=n$, and $j=0$ or $j=m$. In our case, $n=m$. The Bernstein polynomials form a basis for the space of all polynomials in two variables, which means that any polynomial surface is a Bézier surface. Another important property is that they satisfy the *unit partition* condition:

$$\sum_{i=0}^{n}\sum_{j=0}^{n} B_i^n(s) B_j^n(t) \equiv 1 \quad (10)$$

independently of s and t. In the next section it is shown how the control densities are found for a given experimental configuration.

Finding the control densities. Given a trajectory indexed by l, consider all intersections of this trajectory with all the other trajectories. We now will apply an approximation to the *Beer-Lambert* equation in this context. Let $\mathbf{p}_k=(s_k, t_k)$ be the k-th intersection point, and d_k its normalized distance to the previous point $\mathbf{p}_{k-1}$, where $d_0=0$. The attenuation suffered by a gamma ray that crosses a riser section is approximately given by:

$$\sum_{k=1}^{M_l} b(s_k, t_k)\cdot d_k = \delta_l \quad (11)$$

where $b(s_k\ ,\ t_k)$ is the proposed polynomial density, and M_l being the number of intersections in the l-th trajectory. Substituting equation (1) into (2), and rearranging the coefficients, we get:

$$\sum_{i=0}^{n}\sum_{j=0}^{n} b_{ij}\sum_{k=1}^{M_l} B_i^n(s_k)B_j^n(t_k)\cdot d_k = \delta_l \qquad (12)$$

These are the rows of an over-determined system $AX=b$, where X contains the unknowns b_{ij}. The least squares problem is set by the normal equations: $A^TAX=A^T b$.

The tensor product Bézier surface method was developed in two phases: (*i*) the calculation of intersections, iiplemented in C++, producing the matrix A, and (*ii*) the least squares methods was invoked in the MatLab ambient. In Figure 5, is presented the reconstructions from static experiment of a core concentration of catalyst.

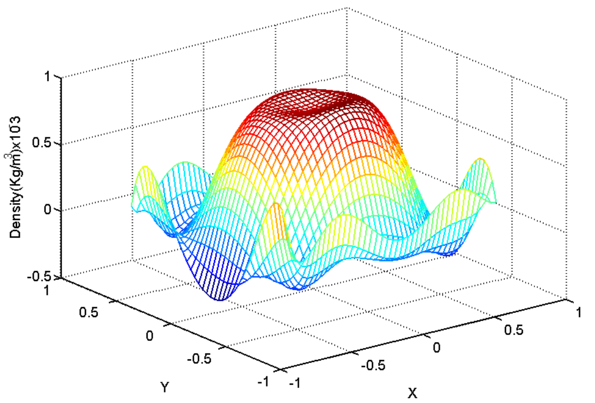

Fig. 5. Tensor product Bézier method with 5 projections.

The tensor Bézier with a higher number of projections show some improvement, as can be seen in Figure 6

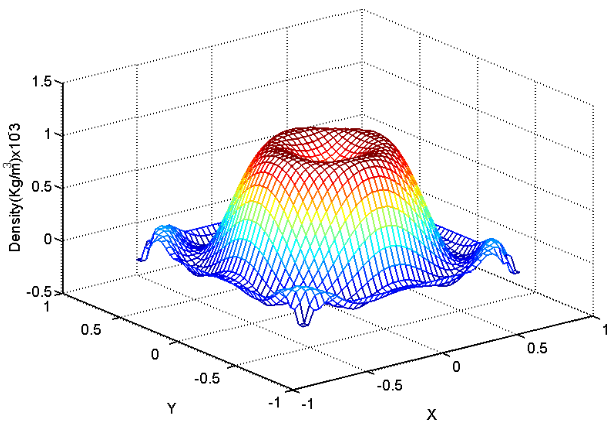

Fig. 6. Tensor product Bézier method with 10 projections.

The possibility of use FBP (Filtered Back Projection), for reconstruction of the catalyst density inside riser, was investigated, and also to make a comparison with Bézier method. Is easy to implement FBP because of the build in Matlab functions, and then to make experiments in order to evaluate the method performance in function of the number of projections. The expected reconstructed density for the experiments is a centralized cylinder trunk of height 0,85 approximately, with a radius of 5 cm. The reconstruction with FBP method for a number of 180 projections is given in Figure 7.

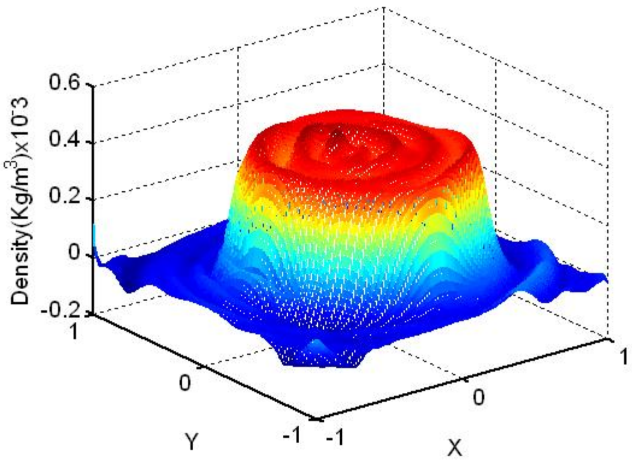

Fig. 7. Reconstruction with FBP method

The reconstruction of the density function by the regular FBP method with only three projections is very poor, with too many artifacts, with the height too low (<0.2). With the same 3 projections, but utilizing a simple interpolation in the sinogram to produce 180 projections Figure 7, the results are much better, with the height still too low (<0.5), and with some artifacts. The tensor product Bézier method applied to 5 projections Figure 5, which is of degree 7, produced a smoother function than the FBP one, but with the height above 0.5, however presenting artifacts near the center, and outside the riser. There is a significant improvement in 10 projections Figure 6, which is of degree 10, with the height close to .85, with fewer artifacts outside the riser, but with a bigger depression near the center. These results show that the tensor product Bézier method is much simpler, but performs favorably in relation to the FBP with few projections.

As an optimal temporal resolution requires a minimum of gamma ray trajectories and a minimum of projections, to improve the reconstruction by using a small number of projections is under investigation. The other factories influencing temporal resolution are: the number of trajectories that is fixed by the Nyquist criterion and the counting time that is selected by the required uncertainty in the intensity measurement.

3. UNCERTAINTY EVALUATIONS

The measurement models were evaluated in the general form

$$y_i = \phi(x_i, a) + \varepsilon_i; \quad \varepsilon \in N(0,\sigma^2) \qquad (13)$$

and the uncertainty associated $u(y_i)$ with y_i is σ and, ε being here a residual error. For a nonlinear function the variance of the parameters is

$$V_a = \sigma^2 (J^T J)^{-1} \quad (14)$$

where J is the Jacobian matrix given in [9]. At first parameter estimator methods were applied and then validation methods are tested, as Monte Carlo simulation that was calculated for nonlinear least square estimator

$$y_{i,q} = y_i^* + \varepsilon_{i,q}; \text{ with } y_i^* = \phi(x_i, a^*) \quad (15)$$

where $\varepsilon_{i,q}$ is determined using a random number generator for normal distribution and a set $z_q = \{(x_i, y_{i,q})\}_{i-1}^m$

The length measured with Equation (3), was used to evaluate the slice volume, of an known object, then combined with density from Equation (7), gives the mass, that multiplied by the gamma beam width provided a mass that was compared to a more extensive data interval to evaluate uncertainty. The combined standard uncertainty was calculated by means of the equation given in [15], that in a compact form can be written as

$$u^2(y) = (\nabla_x f) V_x (\nabla_x f)^T \quad (16)$$

where $\nabla_x f$ are the sensitivity coefficients and V_x the covariance matrix, for simultaneous measurement the uncertainty matrix is $V_y = J_x V_x J_x$, that was calculated, following this notation, according to procedure given by [15].

To calculate the spatial resolution as shown in Figure 2, it was necessary that the gamma beam be expressed as a band-limited function, under such conditions that Fourier transform vanish for a frequency that is higher than the cutoff frequency [16]. A Gaussian function fitted the data best, with a variance σ^2 and it then it was required to apply the lemma: "the Fourier transform of a continuous-time Gaussian function of variance σ^2 is also a Gaussian shape, but with variance $1/\sigma^2$", in order to evaluate their uncertainty [17].

The catalyst radial density distribution was measured and data are presented in Figure , The Fourier representation of the data is given in Equation (8). The Fourier transform of the data provides information on the frequency. The Fourier transform Y of the X is given implicitly by

$$X = AY \quad (17)$$

A is the matrix of the coefficients, according to [15]. The uncertainty matrix associated with estimates y of the values of Y is denoted by V_y, and in a compact form can be written as

$$V_x = A V_y A^T \quad (18)$$

it follows that V_y can be calculated from (11), since A is invertible.

The scanning of the riser is carried out in a spatial frequency and the Fourier function in Equation (8), has a constant frequency w, as it is a static experiment. In flow experiments [18], a Fourier function did not succeed in model catalyst density distribution. For a flow condition riser there occurs frequency content of signal that is locally in time, due to the temporal dependence of the flow. Therefore, the STFT - short time Fourier transform or Windowed Fourier Transform was implemented by means of Matlab functions. Then, a model of the flow was obtained that allows distinguishing a frequency difference and it can be analyzed by changing frequency resolution and time resolution. Monte Carlo simulation was used for uncertainty evaluation but the results are not yet acceptable.

4. CONCLUSION

Modeling in a metrological approach shows the capability of identifying problems and yielding diagnostics from gamma transmission measurements in riser. And certainly a more detailed formulation of the error structures following measurement models is required. Monte Carlo simulation, as a specific technique for model validation and uncertainty calculation seems to be most promising. As is expected, the time dependence of the catalyst flow proves to be the more challenging topic in gamma ray tomographic investigation.

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