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OPTIMIZATION OF MEASUREMENT STRATEGY

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Abstract: Laboratory measurements are an integral part of process and quality management. Quite commonly, variables measured are statistically dependent and thus measurement of one variable is an indirect measurement of the others. Also, measurements are uncertain and thus there always is uncertainty about the quality. In this paper we formulate the optimal measurement scheduling problem of laboratory measurements. In particular, the objective is to minimize a weighted sum of measurement costs and uncertainty about whether the product confirms with given quality constraints or not. This problem is considered in the case when the joint probability density of variables is Gaussian, and when uncertainty evolves according to Ornstein-Uhlenbeck process.

Keywords: quality management, scheduling, optimization

1. INTRODUCTION

In industrial processes, such as in papermaking, the product quality is measured from end product samples in laboratory. This information is typically used for three purposes: firstly to validate on-line sensors, secondly to manage quality parameters not measurable on line –for example strength of paper – and thirdly to decide whether to accept or reject the product batch. Each of the purposes set requirements on how much uncertainty may be tolerated in the quality estimate. It is a common practice to measure all quality parameters at regular time intervals [1, 2]. However, this is costly and may limit possibilities to measure those quality parameters that would be most important for overall uncertainty management and thus for decision support. Earlier work has considered minimizing information measures derived from a covariance matrix [3-5]. We have studied the optimal measurement scheduling such that the measurement cost is minimized under constraints on quality uncertainty, i.e on diagonal elements of the covariance matrix [6].

Every quality parameter has its acceptance limits ("quality pipe") set by the product specification. Quality parameters are typically statistically dependent and thus measuring a subset of them provides information about the others. A measurement schedule describes which of the quality parameters are measured at which time instants. We seek for the optimal measurement schedule over a time horizon such that it minimizes the costs of measurement while minimizing the uncertainty about whether the product quality confirms with acceptance limits or not. This means that when we are close to an acceptance limit, the quality information must be more accurate than far from the limits.

Our quality information dynamics consists of continuous degradation towards the joint *a priori* probability density of quality, and occasional updates when new measurements are made. We describe the degradation with probability density function dynamics, Fokker-Planck equation [7], and the updating with probabilistic description of measurement and by applying Bayesian combination of earlier degraded information and of fresh measurement information. With this information dynamics we are able to assess the quality uncertainty at any time with any measurement schedule.

This paper is organized as follows: Section 2 illustrates the measurement strategy. Section 3 introduces measurement scheduling problem at general form and with only one quality parameter. Section 4 illustrates the idea by studying the effects of model parameters on quality pipe problem of one variable. The general scheduling optimization is further discussed at section 5. And section 6 presents some conclusions.

2. MEASUREMENT STRATEGY

The process industries make use of hundreds of on-line and laboratory measurements to monitor and control the process [2]. Information systems are designed with the aim of supporting the daily decision making about the process and product quality by operators and engineers so that the best practice of operation can be achieved continuously. Measurements, soft sensors and process simulators form the basis for such decision support by reducing the uncertainty about the present state of the process and about its future evolution.

In process industries, such as papermaking, the quality management is commonly based on three level hierarchical measurement structures: accurate but costly and infrequent laboratory measurements, automated quality analyzers sampling more frequently and mimicking laboratory analyses, and indirect but frequent on-line measurements for automatic control. In paper mills, measuring frequency of analyzers is usually once per machine reel, or 1-3 times an hour, whereas laboratory analyses are made at most 3 times a day. These frequencies are to be compared with that paper web is produced continuously at web speed of up to 30 m/s, or 50 tons/h. The decisions supported with all measurement information are process and quality management, special actions, such as grade changes, recovery from process upsets and decisions about rejecting product batches, and configuration of the measurement information system itself.

Industrial operational decision support systems are based on measurement data and on pre-existing process knowledge. However, the uncertainty in the measurements and pre-existing knowledge is not provided by process information system, and thus, uncertainty is rather unfamiliar concept to process operators. This leads to one interesting point of the measurement strategy – decision making is based on the uncertain measurements but decision makers are not so familiar with the concept of uncertainty. Therefore the decision making should be supported with systems such that the uncertainties are described explicitly and consistently, allowing systematic combination of information from several sources. This information derived from the process is used in many ways, but it is poorly known, how and if the operators exploit all the information available. Therefore there has not been systematic work on optimizing the measurement activities. This may lead to situation where some measurements are carried out without purpose, only by habit, and the common practice continues to be to measure all quality parameters at regular time intervals. Obviously this is costly and rigid and may limit possibilities to measure those quality parameters that would be most important for overall quality management and thus for decision support.

However, if end user information requirements, and constraints on uncertainty of the measurements are made explicit, the optimal arrangement of the measurements and decision support system can be determined. Operational decision support systems (ODSS) should reflect the structure of the statistical decision theory [9-11]. Such an ODSS must have description of the state space, the consequence space, decision space, that is, the action alternatives, the pre-existing knowledge, the measurements, the prediction models, the utility function (or the parameters of the heuristic criteria), and the stochastic constraints. With such architecture the ODSS aids an expert or a group of experts in finding decision tasks and helps in experimenting with the decision parameters. [12]

As shown at the figure 1 the end user requirements affect the decision making tasks and thus information needed and the measurements. This leads to our definition about measurement strategy that is optimizing of which and when measurements should be done to get information worth the costs of obtaining it.

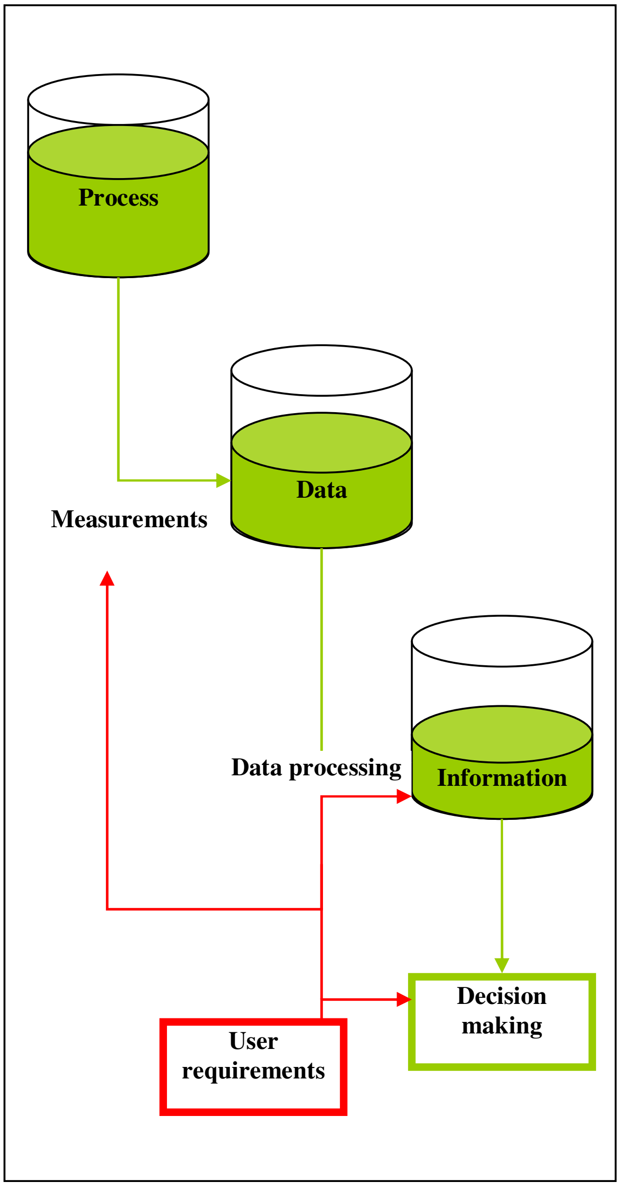

Figure 1. From measurements to data, information and decisions. Red color shows where end user requirements affect and should be taken into consideration.

There are basically three main opportunities to exploit optimization of measurement strategy. Firstly, we optimize the measurements and control actions dynamically for optimal system performance, but this typically leads to complex calculations. The exception is linear-quadratic control in which the control action and measurement selection problems are separable and lead to a measurement policy optimization [8].

Secondly, we may want to constrain the uncertainty of the state information and find the cost-optimal way of satisfying the constraints, see [6], and, thirdly, we may want measurements maximally informative about whether the quality is within specifications or not. This paper concentrates on the last one.

2. OPTIMIZATION OF MEASUREMENT SCHEDULING

This chapter discusses the problem of optimization of measurement scheduling with one quality parameter with constrain of quality pipe. The scheduling problem was first phrased and solved in a linear-quadratic system already by Meier et al [8] in 1967.

It is common industrial practice that every quality variable has its own quality specifications, acceptance limits. Ideally these take into account the measurement uncertainty also, but here we consider, how certain we may be whether the actual product quality – rather than its measured value – conforms with quality specification or not. In this paper we want to search surrounds of the question is it possible to schedule measurements under the quality pipe constraint, figure 2 shows an example about this. At figure black X shows the potential times for sampling and red circles examples one possibility for measurement scheduling.

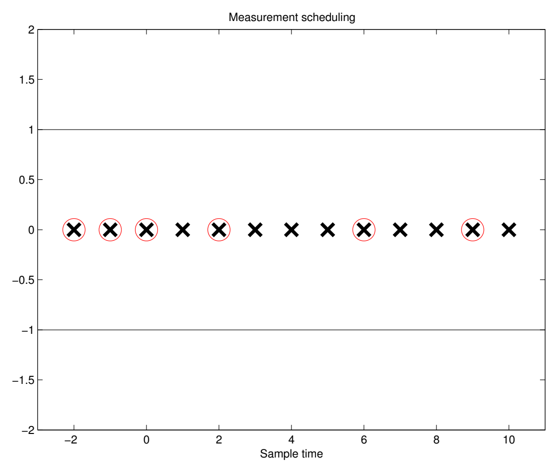

Figure 2. Measurement scheduling with quality pipe constraint. Black X shows the potential times for sampling and red circles one possibility for measurement scheduling.

Let us assume a scalar variable x that at time 0 is normally distributed:

$$f_0(x) = N(x;\mu_0,\sigma_0^2) \tag{1}$$

In general we are interested in if the quality variables $j=1...J$ are within their quality pipes which we choose to be symmetric around zero without loss of generality $[-x_c^{(j)}, x_c^{(j)}]$. Then at any time instant i the probability of variable j being within quality pipe is

$$p_i^{(j)} = \int_{-x_c^{(j)}}^{x_c^{(j)}} f_i^{(j)}(x^{(j)})dx^{(j)} \tag{2}$$

where the distribution is marginalized to variable j from joint distribution.

As the system state is known only through uncertain measurements, only probability p can be given. The measure of uncertainty in knowing whether a quality parameter j is in quality pipe or not is the entropy:

$$S_i^{(j)} = -p_i^{(j)} \log p_i^{(j)} - (1-p_i^{(j)})\log(1-p_i^{(j)}) \tag{3}$$

Where pi is probability of variable being within quality pipe.

Figure 3 shows values for entropy (S_i) with different p_i values.

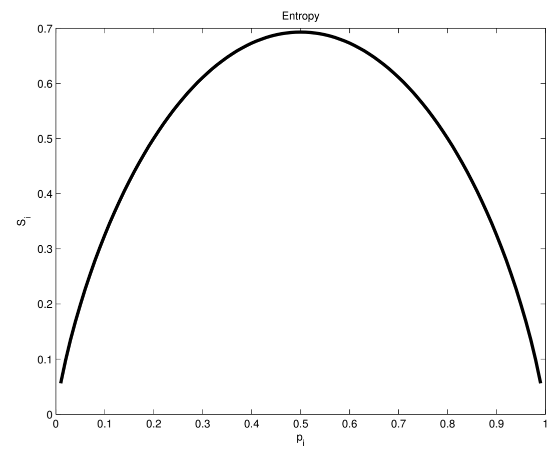

Figure 3. Entropy values

And with (for normal distribution of quality information)

$$p_i^{(j)} = \frac{1}{2}\left(erf\left(\frac{\mu_i^{(j)} + x_c^{(j)}}{2^{1/2}\sigma^{(j)}}\right) - erf\left(\frac{\mu_i^{(j)} - x_c^{(j)}}{2^{1/2}\sigma^{(j)}}\right)\right) \tag{4}$$

We have an option to make a measurement about x at any time instant i. The measurement vector y is uncertain representation of x and described with a normal distribution:

$$f^{(meas)}(y_i \mid x) = N(y_i; x, \Sigma_{meas}) \tag{5}$$

We are interested in the case in which the information about state degrades according to Ornstein-Uhlenbeck (OU) process with linear deterministic dynamics described by matrix B and stochastic part by diffusion matrix D. This means that if no measurements are made, the information degrades towards normal distribution with mean x_{ap} and covariance matrix $DB^{-1}/2$. After a time step of Δt and no measurement made, the normally distributed information about quality x remains normally distributed, and the distribution parameters are recursively calculated as:

$$\begin{aligned} f_i(x) &= N(x, \mu_i, \Sigma_i) \\ \mu_i &= x_{ap} + \exp[-B\Delta t](\mu_{i-1} - x_{ap}) \\ \Sigma_i &= \Sigma_{i-1}\exp[-2B\Delta t] + \frac{1}{2}DB^{-1}(1-\exp[-2Bt]) \end{aligned} \tag{6}$$

A special case of OU process is random walk, corresponding to limit B tends to zero. In this paper we limit ourselves to one variable. Then the state information without measurement develops as:

$$\begin{aligned} f_i(x) &= N(x;\mu_i,\sigma_i^2) \\ \mu_i &= x_{ap} + \exp(-B\Delta t)(\mu_{i-1} - x_{ap}) \\ \sigma_i^2 &= \sigma_{i-1}^2 \exp(-2B\Delta t) + \sigma_{ap}^2 (1 - \exp(-2B\Delta t)) \end{aligned} \tag{7}$$

and if measurement are made and result y_i, is obtained then the information is

$$f_i(x) = N\left(x; \frac{\mu_i \sigma_{meas}^2 + y_i \sigma_i^2}{\sigma_{meas}^2 + \sigma_i^2}, \left(\sigma_{meas}^{-2} + \sigma_i^{-2}\right)^{-1} \right) \tag{8}$$

Our goal is to minimize information cost, that is, that of making a measurement to improve information and the one related to operating at with poor information

$$G_N\left[f_0(x)\right] = G_N\left(\mu_0,\sigma_0^2\right) = \min_{\{m_i\}_{i=1}^N} E\left\{ \sum_{i=1}^{N} \alpha^{i-1}\left(h(m_i) + g(S_i)\right) \middle| f_0(x) \right\} \tag{9}$$

with convention of m=0 meaning no measurement, h is the cost of measurement with $h(0)$ being 0, and m=1 making the measurement at cost $h(1)=h$. g is a monotonously increasing function, such as $g(x)=x$, or $g(x)=exp(x)$. Expectation is be calculated with respect to information at time $t=0$.

Now, let us consider the one-step ahead problem with $g(x)=x$.

$$\begin{aligned} & h + b\sum_{i=1}^{N} E\left\{S_i\right\} \\ & \Rightarrow h < b * \left(E\left\{S_N^{(no)}\right\} - E\left\{S_N^{(yes)}\right\} \right) \end{aligned} \tag{10}$$

where b is a factor scaling entropy cost to be comparable with measurement cost, $S_N^{(no)}$ meaning entropy at time N when no measurement is done and $S_N^{(yes)}$ is corresponding entropy when measurement is done at time N and expectation is taken with respect to future measurements.

We want to make next measurement when achieved information is more than the cost of the measurement. Figure 4 show how the two entropy terms depend on N.

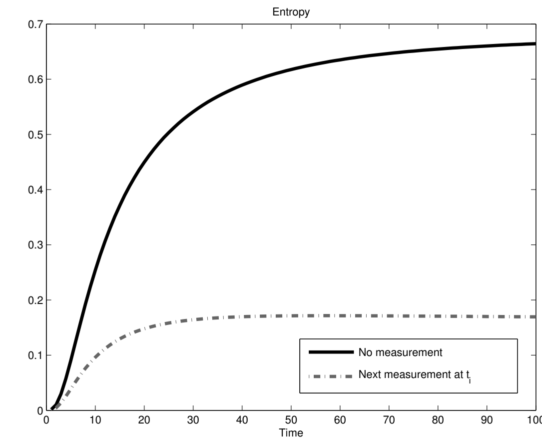

Figure 4. Entropy with no measurement and measurement at t_i *with values x_0 = 0, σ_0 = 0.04, D = 0.03, x_c = , x_{ap} = 0, σ_{ap} =1.5 and σ_{meas} = 0.04.*

When calculating the entropy, initial estimate (x_0) and variance (σ_0), expected (*a priori*) estimate (x_{ap}) and variance (σ_{ap}) with measurement uncertainty (σ_{meas}) is used and p is calculated with cumulative distribution function. Uncertainty between the measurements evolves according to Ornstein-Uhlenbeck process.

4. PARAMETER EFFECTS IN MEASUREMENT SCHEDULING

This section studies the effects of initial information and model parameters on quality pipe problem of one variable.

Deriving from Eq. (10) we want to minimize the following:

$$t_{meas} = \arg\min_N \left(\frac{h + b\sum_{i=1}^{N} S_i}{N} \right) \tag{11}$$

Where h is cost of making the measurement, t_{meas} is time for measurement, b is a factor scaling entropy cost to be comparable with measurement cost, N is time index and sum is difference of entropies. Quality pipe is centered at 0.

Figure 5 shows how parameter D values (0.01, 0.03, 0.1 and 0.3) affect t_{meas} (60, 20, 6, 2 corresponding). Parameter D is the symmetric diffusion matrix for the OU process.

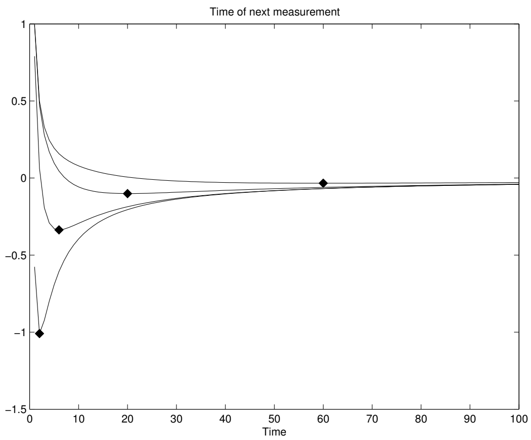

Figure 5 Effect of Parameter *D* to t_{meas}

Figure 6 shows affect of initial estimate x_0 to t_{meas}. Values for x_0 is 1, 0.8, 0.6, 0.4, 0.2, 0 and respectively for t_{meas} 1, 1, 12, 30, 53 and 60. When initial estimate is near the edge of the quality pipe then entropy is high and next measurement should be done quite soon.

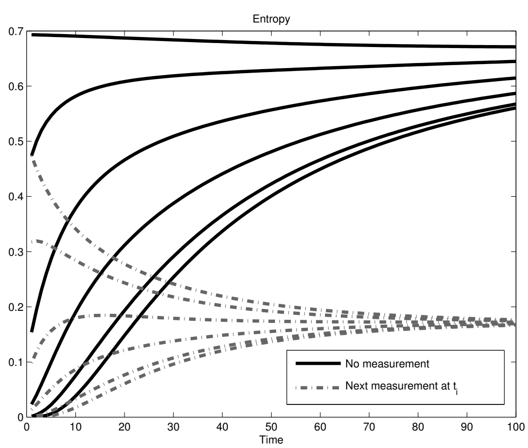

Figure 6 Effect of initial estimate (x_0) to t_{meas}

It is quite simple to calculate the next t_{meas} when measurement at t_0 is made and at given probability we can tell if measurement is inside the quality pipe or not. On the other hand after the measurement at t_{meas} everything depends on the measurement value, so instead of one value we have to consider all the possible values and this leads quite complex calculations. If the process is known very well the distribution may be used but this also leads to complex simulations.

Specifications for quality is usually presented one variable at the time so entropy (S) and $p_i^{(j)}$ can be handled also by one quality variable at the time although there might be correlation between the quality variables.

5. DISCUSSION

This discussion chapter presents some ideas how to solve this measurement scheduling problem. This problem will easily explode, because of the possible variations at measurement results. Different solution possibilities for this problem will be searched amongst Markov chains, partially observable Markov decision process (POMDP) and with dynamic programming. These provide some tools against the curse of dimensionality.

Markov chain is a stochastic process with the Markov property meaning that, given the present state, future states are independent of the past states. Future states will be reached through a probabilistic process instead of a deterministic one. At each instant the system may change its state from the current state to another state, or remain in the same state, according to a certain probability distribution. [13]

Partially Observable Markov Decision Process (POMDP) is a generalization of a Markov Decision Process. A POMDP models an agent decision process in which it is assumed that the system dynamics are determined by an MDP, but the agent cannot directly observe the underlying state. Instead, it must infer a distribution over the state based on a model of the world and some local observations. The POMDP framework is general enough to model a variety of real-world sequential decision processes. Applications include planning under uncertainty in general. An exact solution to a POMDP yields the so-called optimal action for each possible belief over the world states. The optimal action maximizes (or minimizes) the expected reward (or cost) of the agent over a possibly infinite horizon. The sequence of optimal actions is known as the optimal policy of the agent for interacting with its environment. [14,15]

Dynamic programming is a method of solving problems exhibiting the properties of overlapping sub problems and optimal substructure that takes much less time than naive methods. [16]

Advantages of measurement optimization and scheduling include the process of acquiring the values for uncertainties and quality pipes of different variables. Although this may be time consuming it is rewarding itself because the knowledge about the process grows.

6. CONCLUSIONS

In this paper we have discussed the optimal measurement strategy optimization problem of laboratory measurements under the constraint that variables has quality pipe. This problem is considered in the case when the joint probability density of variables is Gaussian, and when uncertainty evolves according to Ornstein-Uhlenbeck process. This paper has concentrated mainly presenting the scheduling problem.

Formulation of the problem is presented at general form and with the case of one quality variable. The effects of model parameters on quality pipe problem of one variable have been considered. Some ideas for solution is thought over.

This paper is mainly research plan trying to formulate a problem and finding some possible solutions for it. We hope that in the future we have solution for this measurement scheduling (with quality pipe constraint) problem and this paper serves as our guide.

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