

BAYESIAN NETWORK MODEL AS AN OVERALL IMAGE QUALITY MEASUREMENT SYSTEM

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Abstract: This paper discusses a measuring system for overall image quality with a jury assessment as reference. The problem of combining subjective assessments and objective measurements is studied within the framework of Bayesian statistical theory and the Bayes' network. The ideas are demonstrated with a subjective assessment test case and simulations.

Keywords: Bayesian models, image quality, qualimetry.

1. INTRODUCTION

Human perception of product quality plays the key role in marketing and product development. Subjective quality experience affects the value of the product. Obtaining meaningful subjective quality data is tedious and expensive. It requires a jury of people and the procedure is complex and time consuming. Therefore, models are being searched to replace the subjective assessments by instrumental measurements. Typically, a jury is a reference for such instrumental quality assessment system. In this approach, the model linking subjective quality and instrumental measurements is identified with data on subjective human assessments and objective instrumental measurements on the same samples.

The measurement of high level image quality is widely considered as weakly defined [1]. This is because overall image quality lacks specific standards linking quality to physical sciences. The image quality measurement at present lacks the empirical proofs in the theory of relations and completeness. I3A [2], as an organization, is collecting the research results in order to provide a solid foundation for the quality measurement of images.

The theory and current research in the area of image qualimetry in [2-4] has concentrated on finding the subjective attributes of images that correlate with instrumental measures, or attributes that are based on the function of the human visual system. The overall quality of a printed image, for instance a poster or a picture in a magazine, is the combined effect of various objective and subjective factors related to image capturing, image processing, printing process and paper. In the area of print quality measurement, some relations have been successfully established between physical measures and subjective assessments, such as Heliotest and unevenness of paper [5, 6]. In this study, we have concentrated on modeling the

assessment of digital images and thus limited the number of factors affecting the overall quality.

It is unlikely that overall image quality could be measured with a single objective measurement or that human vision could be simulated perfectly. That is why we have modeled the image quality measurement system as a top-down Bayesian model that simulates the subjectivity and stochastic variability of human assessments in the general quality measurement. The Bayesian statistics makes it possible to view the overall image quality as a probability distribution of states in a discrete model or as probability density functions in the continuous state models.

The idea in the constructed Bayesian model is to infer about the high level quality when instrumental measurements are given as input. To achieve this, first the model has to be identified with assessments obtained from a group of human evaluators known as a jury. In this paper, the evaluators in a jury make independent assessments. There are limitations and requirements for a jury to be a valid reference. The human assessments are often considered poor, since attributes and assessment scales are not unambiguous and hence the results tend to have significant uncertainty [1]. The subjectivity of human assessments may also cause the decision of a jury to seem irrational, although each juror was rational [7]. Furthermore, even a single juror's assessments may have transitive irrationality leading to circular preference (for example, $A > B$, $B > C$ and $C > A$, would form a cycle $A > B > C > A$) [8]. The investigation of circular preferences is one way to assess the uniformity of a jury assessment, or the assessments of a single juror [9]. The lack of rationality increases the uncertainty in jury assessment and thus decreases the reliability of the results.

The overall image quality is a subjective measure and as such, the scaling is difficult to define. That is why a common scale for psychological measures is the just noticeable difference (JND), which measures the perceptual continuum of an attribute [3]. The calculation of JND values is based on the probability of preference in a certain attribute between test samples. The JNDs can be extended to JND increments, which is a metric for subjective differences as a function of objective differences [3]. This is a rather limited view, because the image quality is experienced as multidimensional and usually the relative importance of subjective attributes varies between the evaluators.

According to the theory in [4], there are three choices for the use of a reference image in the measurement of image quality: full-reference, no-reference and reduced reference. In the full-reference method, measurement is based on the comparison of the reference image and the observed image. In the no-reference method, the quantities of interest are measured from the observed image only. The reduced reference is based on the comparison of the features of reference and sample image. A Bayesian model can be constructed with any of the mentioned reference methods. The model for overall image quality measurement introduced in this paper will be identified first with full-reference and reduced-reference instrumental measurements and subjective assessments that are obtained from a restricted sample set. Once the model is identified, it is supposed to operate as a no-reference measuring system.

This paper is organized as follows. Section 2 first introduces the concept of Bayesian network modeling, inference and validation, and discusses its benefits. In Section 3, the Bayesian model is applied as an overall image quality measurement system and the test case for model identification is described. Section 4 concludes the paper with discussion and proposals for future work with possible model development.

2. CONSTRUCTION OF BAYESIAN MODEL

Our measurement model of overall image quality resembles the human quality perception process. The model constructed is a four-layered Bayesian network that consists of high level image quality, low level image quality, perceptual quality elements (PQEs) and instrumental measurements. Figure 1 shows schematically the structure of the model. The arrows indicate the statistical causality of the model. The construction of the model consists of selecting the structure, i.e. how the arrows between the blocks are connected, and identifying the statistical dependence between the blocks as conditional probabilities. As there will be only a single path between any two nodes, the Bayesian model is considered as a polytree graph [10].

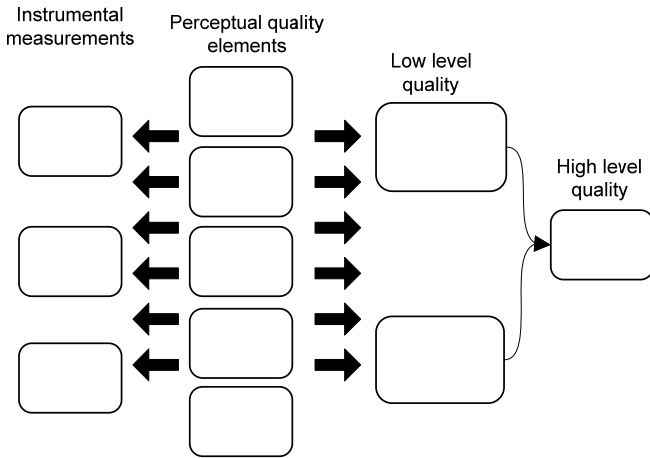


Figure 1. An example of the image quality measurement system modeled as a Bayesian network.

The foundation of the model is the PQE level. The PQEs are assumed to be the attributes of the image that the humans can directly assess and that their assessments – in spite of the non-physical nature – are objective, i.e. the jury would largely have consistent opinions about them. Hence PQEs represent the “ground truth” about perceived quality and thus determine statistically both the instrumental measurement values and the low and high levels of subjective image quality.

The high level quality is its own layer that contains a single node describing the overall quality of the image. Unlike the other layers of the model, the high level quality is expected to be context dependent. Low level quality consists of the attributes naturalness and usefulness which are abstract aspects of the image and together compose the overall quality [11]. The PQEs and instrumental measurements included in the model may be changed according to the image assessment task in question. The model can be identified when observations from each layer are available: instrumental measurement values together with the subjective assessments of PQEs, low level and high level quality attributes.

The key idea in modeling the quality assessment with a Bayesian network is that once the model has been identified, it is possible to consider the overall image quality and other attributes as probability distributions. In particular, any evidence about a node state can be propagated through the network so that the probabilities of the states of each node are updated. Obviously, obtaining evidence of instrumental values through measurements and then inferring about the high level quality is the main intended use. Our choice of using a probabilistic modeling framework for this purpose is in accordance with describing measurement uncertainty in more traditional measurement systems.

Other models, such as neural networks or fuzzy logic, are not as suitable for our study as the Bayesian network model. Neural networks do not explain the reasoning as parameters that are linked to the real-world [12]. They are widely used in effective classification problems where optimizing the final classification result is the main objective and the intermediate stages are not of interest. Fuzzy logic solutions are suitable for continuous process control systems as they are easy to understand, use and update, and they enable causal reasoning through inference [12]. However, fuzzy logic solutions, as well as neural networks, lack the ability of Bayesian networks to model the subjectivity of the assessments as randomness, and to convey the resulting uncertainty through the model using the straightforward Bayesian principles.

2.1. Inference with the Bayesian model

The joint probability of the states in the directed Bayesian model is:

$$p(x) = \prod_{k=1}^K p(x_k | pa_k), \quad (1)$$

where K is the number of nodes in the model, x_k is the examined node, and pa_k are the parent nodes of x_k [10]. For nodes without parents, a priori information is used for the distribution of states of the examined node. In our model, we assume that d-separation property holds so that the

model is a directed acyclic graph with causal directions [10, 13]. The graph of Figure 1 consists of four layers and can be written – neglecting the detailed structure within layers – as:

$$p(x_{inst}, x_{PQE}, x_{LL}, x_{HL}) = p(x_{PQE}) \cdot p(x_{inst}|x_{PQE}) \cdot p(x_{LL}|x_{PQE}) \cdot p(x_{HL}|x_{LL}). \quad (2)$$

When the model is being used and experimental evidence on instrumental measurements is obtained, i.e. $x_{inst} = x_{inst}^{(e)}$, the PQE level probability is derived through Bayes formula as:

$$p^{(e)}(x_{PQE}|x_{inst}^{(e)}) = \frac{p(x_{inst}^{(e)}|x_{PQE})p(x_{PQE})}{\sum_{x_{PQE}} p(x_{inst}^{(e)}|x_{PQE})p(x_{PQE})}. \quad (3)$$

Then information about the (high level) *overall image quality*, x_{HL} , is the corresponding conditional probability distribution:

$$\sum_{x_{LL}} \sum_{x_{PQE}} p(x_{HL}|x_{LL}) \cdot p(x_{LL}|x_{PQE}) \cdot p^{(e)}(x_{PQE}|x_{inst}^{(e)}). \quad (4)$$

For instance, if we had evidence on attributes in the PQE level, the child nodes in the instrumental level would not give any extra information about high level quality, because of the d-separation of the model [10].

In general, the real technical measurements give continuous values as a result. The relation between continuous values in instrumental level and PQE level can be written as:

$$p^{(e)}(x_{PQE}|x_{inst}^{(e)}) = \frac{f(x_{inst}^{(e)}|x_{PQE})p(x_{PQE})}{\sum_{x_{PQE}} f(x_{inst}^{(e)}|x_{PQE})p(x_{PQE})}, \quad (5)$$

where f denotes the probability density function.

2.2. Bayesian model identification

The directed edges of a Bayesian network describe the probabilistic relations between the nodes [10]. The edge structure in a Bayesian model is identified from the relations between attributes in an identification data set. Mutual information and Pearson correlation are examples of methods for choosing edges between nodes in a Bayesian model.

The benefit of mutual information is that it applies also when the relations are highly nonlinear. Mutual information between random variables X and Y is calculated from probabilistic data as [14]:

$$I(X; Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}, \quad (6)$$

where $p(x,y)$ is the joint probability distribution function (pdf) of X and Y, and $p(x)$ and $p(y)$ are the marginal pdfs. In this study, we use the natural logarithm and thus, the unit of mutual information is *nat*. The mutual information is considered as the reduction of uncertainty of Y due to the knowledge of X [14]:

$$I(X; Y) = H(Y) - H(Y|X), \quad (7)$$

where $H(Y)$ is the entropy of Y and $H(Y|X)$ is the conditional entropy of Y given that X is known [14]. The mutual information is symmetrical, i.e. $I(X; Y) = I(Y; X)$. As can be seen from Equation 7, the mutual information, $I(X; Y)$ is theoretically at maximum when the conditional entropy $H(Y|X)$ is zero, and the mutual information equals entropy. The lower bound for mutual information is zero, because if X and Y are statistically independent, then $H(Y|X) = H(Y)$ and hence $I(X; Y) = H(Y) - H(Y) = 0$.

Pearson correlation is calculated as:

$$r_{x,y} = \frac{\sum xy - (\sum x)(\sum y)/n}{\{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]\}^{1/2}}, \quad (8)$$

where n is the number of samples in the random vectors x and y [15]. Pearson correlation corresponds to linear correlation coefficient and it can only have values between -1 (full negative correlation) and 1 (full positive correlation). Pearson correlation values near zero mean that there is no linear correlation between x and y .

Using either of the methods presented above, the dependency between each pair of attributes is examined and this information is used to establish the relevant interconnections as edges. The model parameters are identified as conditional state probabilities according to the identification data. For instance, it is typical in an identification data set that the *overall quality* is judged as low, if both *usefulness* and *naturalness* are judged as low. The conditional probability distribution of a node is estimated at all combinations of states of parent nodes by simply counting the observations in the identification data. If no observations of a combination of parent's states exist, the state of the node is assumed evenly distributed according to the maximum entropy principle.

2.3. Complexity of Bayesian model

The model description does not limit the number of edges chosen for the model, but as the number of edges increase, the model becomes more complex. Greater complexity means that a larger jury assessment data is needed for the model identification.

The complexity of joint distribution of a node grows exponentially in proportion to the number of its parent nodes [10]. In general for a constant number, N , of discrete states in each node, the number of possible state combinations of conditional probabilities is:

$$S = N^{M+1}, \quad (9)$$

where M is the number of parent nodes. For example, if *usefulness*, *naturalness* and *overall quality* are discrete nodes with N states in each, the number of state combinations in the conditional probability of *overall quality* given *usefulness* and *naturalness* is $S = N^{2+1}$. The number of jury assessments needed for model identification is in proportion to the maximum number of state combinations of the conditional probabilities in a discrete model.

2.4. Evaluation of structure

To evaluate the goodness of fit of the model, it is useful to measure the error between the marginal probabilities of

the identified model and the simulated model. The identified model is the original Bayesian network modeled according to the human assessment data and the simulated model is obtained by re-identifying the model with simulation data that is a set of random samples drawn from the identified model. The error is calculated from the difference of state probabilities between marginal distributions of the original (O) and the simulated (R) model. A convenient and widely used measure is the root mean square error (RMSE):

$$RMSE = \sqrt{\frac{1}{T \cdot G} \sum_{t=1}^T \sum_{g=1}^G (R_{tg} - O_{tg})^2}, \quad (10)$$

where t is the index of a node and g is the index of a state. For each marginal probability of a node t , there are S_t states. Thus, there are a total of

$$F = \sum_{t=1}^T S_t \quad (11)$$

parameters in the model for error comparison as one parameter corresponds to a probability of a state of a single node.

A complex Bayesian model can be validated through inspecting the Markov blankets of the model. A Markov blanket for a node is the part of the Bayesian network that consists of the node itself, its parents, its children and the parents of the child nodes, i.e. co-parents [10]. The extended Markov blanket is the part of the network that covers the Markov blankets for all nodes in a single Markov blanket of an inspected node [16]. The extended Markov blanket contains all the necessary information for backpropagating the values of a single node. If the result of the backpropagation differs significantly from node value obtained through measurements, the measurement may be considered faulty [16]. In relatively small models, such as in this study where the Markov blanket covers the most of the network, it is more feasible to check error possibilities through the backpropagation of the whole network. As the Bayesian network complexity is presumed to increase by new layers or nodes, the extended Markov blankets become useful because they simplify the computation of marginal distributions for the node states.

3. RESULTS

To demonstrate the idea of Bayesian network modeling in image quality assessment, we conducted a small visual assessment test with one image content. The model presented in Figure 1 was identified through jury assessments that consisted of high level, low level and PQE attributes for modified images. At first, the instrumental measurement layer consisted of image manipulations that simulated the results obtained with real measuring devices. For testing the validity of continuous measurement, the simulated measurements were later replaced by algorithmic measurements.

The edges in the model between low level quality and high level quality are based on the research conducted in [11], where it is suggested that overall image quality can be explained with naturalness and usefulness attributes. The rest of the edges were chosen according to the mutual information between the attributes on successive quality

levels, i.e., between simulated instrumental measurements and PQEs, and between the PQEs and low level quality. The PQEs applied in our test were five attributes that had frequently been mentioned in previous studies [9, 17], namely sharpness, brightness, colorfulness, graininess and clarity. The meaning of each attribute was shortly explained to each evaluator so that the variance of subjective understanding could be decreased.

3.1. Experiment setup

In the test case, a digital image shot in a studio was modified by three methods: HSV saturation adjustment, low-pass filtering and noise addition. The degree of these modifications simulated the instrumental measurements in the Bayesian model. We used three distinct degrees of modification for each method: no modification, mild, and moderate level. The combination of all modified images was used in the subjective assessment test, that is, the number of images was 27. Each image was assessed with respect to eight attributes (5+2+1) on a scale from 1 to 5, one attribute at a time. In practice this means that each subject was asked to label the 27 images with the grades 1-5 eight times. The scale approach was chosen over rank ordering or paired comparisons, since it is faster and it provides a rough quantitative assessment of the attribute value [8]. The image used in the test is shown in Figure 2.



Figure 2. Studio image used in the Bayesian network identification test.

The Studio image had been designed to cover the subjective and objective factors as much as possible. Consequently, it contained several detailed objects that required evaluator's attention. Each evaluator weights differently the components of the image while assessing certain attributes, which is a reason that may cause irrationality in jury decision. The irrationality is a source of uncertainty in the model identification and therefore it may cause modeling error. The human assessments were done using a monitor display and a graphical user interface built for MATLAB.

3.2. Modeling results

The edges between the instrumental level and PQE level were defined by the largest mutual information values between a node in the instrumental measurement level and two nodes in PQE level, thus the number of edges in the model was constrained to two. The edges leaving the PQE

level and directing to either low level node were also allocated according to the largest mutual information. The attributes were combined to a model shown in Figure 3.

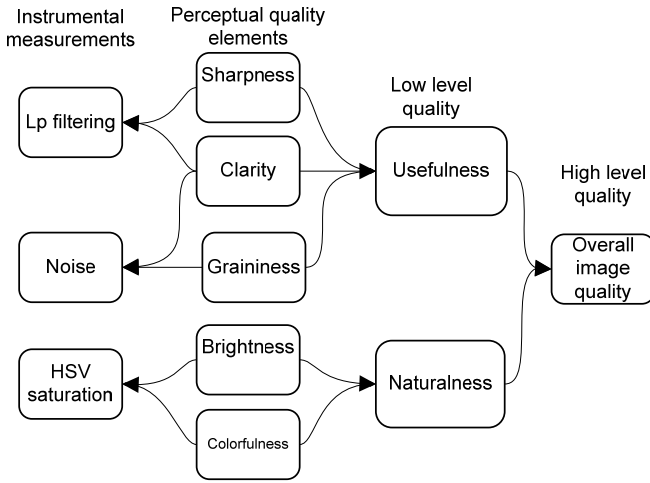


Figure 3. The Bayesian model constructed according to the mutual information results computed from the test case data.

Figure 3 shows that as the edge decision rule is the mutual information, according to the human assessments, the model is divided into two branches. The mutual information and Pearson correlation values between the attributes related in the model are shown in Table 1.

Table 1: Mutual information and Pearson correlation values for nodes in Bayesian model identification.

		I as nats	$r_{x,y}$
HSV saturation	Brightness	0.0755	0.0172
	Colorfulness	0.1825	-0.2256
Low-pass filtering	Sharpness	0.5633	-0.8392
	Clarity	0.2101	-0.5349
Noise	Graininess	0.3343	-0.6085
	Clarity	0.1989	-0.4942
Usefulness	Sharpness	0.5300	0.7660
	Graininess	0.1134	0.3081
	Clarity	0.4340	0.7374
Naturalness	Brightness	0.1550	0.2648
	Colorfulness	0.1907	0.4776
Overall quality	Usefulness	0.2662	0.5899
	Naturalness	0.1649	0.4252

The values of Pearson correlation vary between -1 and 1. The theoretical maximum value of mutual information is the entropy of a uniform distribution of a node with fewer states [14], as instrumental measurement nodes have three states and all other nodes have five states. In the model identified according the Pearson correlations, the structure is very similar to model identified with mutual information. The only difference is that the edge from *brightness* to *HSV saturation* is changed to edge from *clarity* to *HSV saturation*. In that case, the *brightness* attribute would not be

directly utilized in inference from instrumental level to high level quality.

3.3. Results of structure evaluation

The goodness of fit of the model was tested by drawing random samples from the original model, re-identifying and obtaining the simulation model, and evaluating the error between marginal distributions of all nodes between the two models. Data sets of 10, 100, 300, 500, 800, 1000, 1500, 2000 samples were drawn from the identified model separately. The sizes of the data sets correspond to approximately 1, 4, 11, 19, 30, 37, 56 and 74 human evaluators respectively. The data sets were drawn 100 times from the original model and for each time, the model was re-identified and the marginal distributions of the nodes were evaluated.

The RMSE between the original and the simulated model is shown in Figure 4. The RMSE was evaluated for each number of samples 100 times to diminish the effect of variance between sample sets.

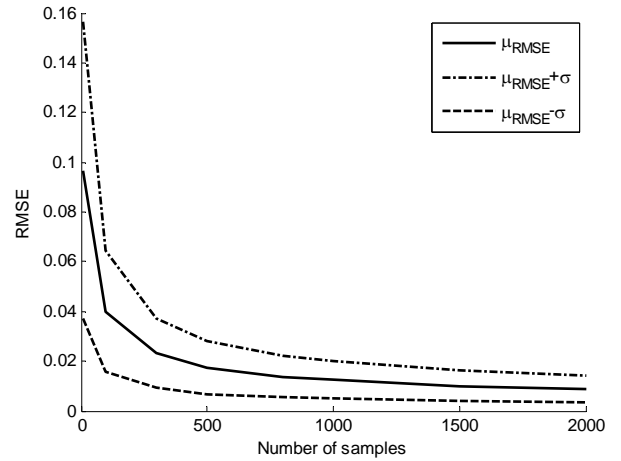


Figure 4. The RMSE between the original model and the simulated model. The number samples, for which the RMSE was calculated, were 10, 100, 300, 500, 800, 1000, 1500 and 2000.

According to Equation 11, there are 49 state parameters in the simulated model. The simulated model may be considered adequate with more than 500 samples.

3.4. Modeling with the continuous measurements

To better model the instrumental measurements, algorithmic measurements were conducted for all 27 images. Algorithms for evaluating blur [18], contrast [19] and noise [20] as no-reference measurements were utilized. For defining the relations between PQEs and algorithmic measurements, the continuous results were first quantized to five states. The relations of these measurements to the PQE level attributes as mutual information are shown in Table 2.

Table 2 implies that if the number of edges directed to each instrumental measurement node is constrained to two, there would be no edges leaving *brightness* or *colorfulness* nodes, because they have the smallest amounts of mutual information in relation to *noise*, *contrast* and *blur* measurements. This could be solved by further developing the measurement algorithm or increasing the number of edges.

Table 2: Mutual information as nats between algorithmic measurements and PQEs.

	Noise	Contrast	Blur
Sharpness	0.4303	0.5068	0.3486
Brightness	0.0950	0.0553	0.0753
Colorfulness	0.0476	0.0847	0.0462
Graininess	0.3919	0.0772	0.1954
Clarity	0.3496	0.1859	0.2556

A simple example of the efficiency of inference with the continuous measurements is shown in Figure 5.

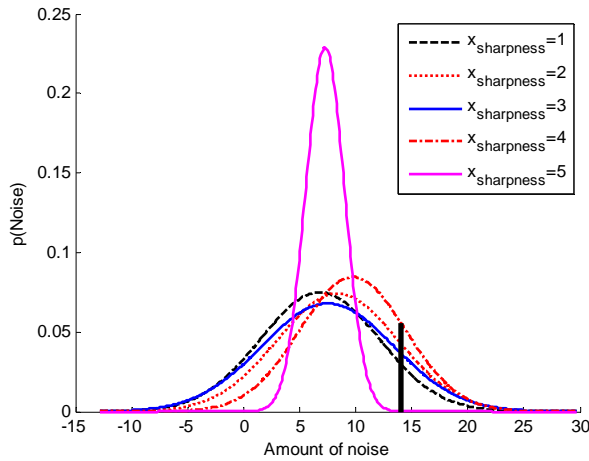


Figure 5. The pdfs of algorithmic noise at each state of sharpness attribute.

According to Equation 5, we may evaluate the maximum likelihood estimate for the continuous result given the observations from the PQE level. The probability distributions of the amount of noise at each state of sharpness are presumed to be Gaussian distributions whose mean values and standard deviations have been calculated from the human assessment data.

The pdf curves in Figure 5 can now be used to deduce the most likely state of sharpness, given that the noise level has been measured. For example, if the noise was algorithmically measured to be 14 (black vertical line in Figure 5), the probabilities of the states 1, 2, 3, 4 and 5 of sharpness are 0.1806, 0.2588, 0.2224, 0.3375 and 0.0007, respectively, and thus the most likely state of sharpness is 4. Evaluation of the state probabilities of more than one parent is calculated the same way, but the number of continuous functions increases exponentially as described in Equation 9, i.e. if the number of parents is three and each parent may be in five states, there would be $5^{3+1} = 625$ continuous functions. This would greatly increase the amount of data needed for identification of the distributions.

4. DISCUSSION

This paper discussed Bayesian network models. The construction, identification, inference and structure evaluation were the main subjects. We have shown the advantages and limitations of Bayesian modeling and

supported the facts with a simple test case conducted through human assessments and algorithmic evaluation of certain image quality characteristics. The Bayesian model for overall image quality is an example foundation for a construction of a model that combines the continuous instrumental measurements and the subjective assessments.

The measurement of image quality is a weakly defined soft system, because in our case, the measurement system involves the uncertainty of human factor and there is no solid theory in that area [21]. As a soft system, the predictive validity of image quality measurement is limited if the circumstances of system change, i.e. if the context of image changes so that the relative importance of naturalness and usefulness is changed in the assessments of most of the jurors [21]. If the system is continuously updated by human decisions, the system is self-aware. The self-awareness is an undesired feature in image quality measurement systems since the aim of this research is to lessen the dependency of the human factor. The context effect will probably have to be decided by a human expert, since an automatic context recognition system of images is an extremely difficult task to design or implement for a computer.

Due to the context dependency of image quality assessment, the process has to be worked in one image context class at a time. Nevertheless, the system model seems promising for the development of overall image quality measurement. Our current model requires further examination and human assessment tests but it offers a solid conceptual foundation for future research. Once a valid model for image quality measurement is tested and implemented, similar approach may be applied to video quality measurement as well.

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