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CALIBRATION UNCERTAINTY ESTIMATION FOR INTERNAL AERODYNAMIC BALANCE

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Abstract: Aerodynamic balances are employed in wind tunnels to estimate the forces and moments acting on the model under test. This paper proposes a methodology for the assessment of uncertainty in the calibration of an internal multi-component aerodynamic balance. In order to obtain a suitable model to provide aerodynamic loads from the balance sensor responses, a calibration is performed prior to the tests by applying known weights to the balance. A multivariate polynomial fitting by the least squares method is used to interpolate the calibration data points. The uncertainties of both the applied loads and the readings of the sensors are considered in the regression. The data reduction includes the estimation of the calibration coefficients, the predicted values of the load components and their corresponding uncertainties, as well as the goodness of fit.

Keywords: internal strain-gage balance, calibration uncertainty, wind tunnel tests.

1. INTRODUCTION

The staff of the Aerodynamic Division of the Institute of Aeronautics and Space, Brazil, has been concerned with the quality of the data originating from aerodynamic tests. Following international recommendations based on the Metre Convention, several studies have been conducted in order to improve the metrological reliability of measurements, tests and calibration procedures carried out in the Subsonic and Transonic Facilities. One of these projects is the development of methodology for the assessment of uncertainty in the aerodynamic loads acting on the test article.

The instrument used to estimate the aerodynamic loads is the aerodynamic balance, whose calibration uncertainty contributes significantly to the overall uncertainty in wind tunnel testing.

International organizations, such as the American Institute of Aeronautics and Astronautics (AIAA), have been exchanging information and collaborating on the development of a balance calibration uncertainty methodology, but this has been only partially addressed **[1]**. The aim of this study is to present a methodology for the uncertainty estimation of internal balance calibration, according to international recommendations **[2]**.

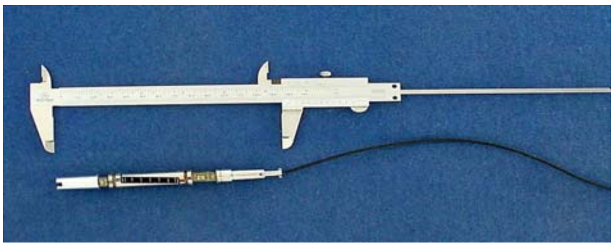
1.1. Aerodynamic loads

The terminology employed for designating the aerodynamic load components is: axial force (*AF*), side force (*SF*), normal force (*NF*), rolling moment (*RM*), pitching moment (*PM*) and yawing moment (*YM*).

1.2. Multi-component aerodynamic balances

Basically, there are two kinds of balances: internal and external. The former is designed to fit within the test article and the latter carries the loads outside the tunnel before they are measured (Fig. 1).

a)



b)

Fig.1. a) External balance. b) Internal balance.

The balance measures the loads by using strain-gages arranged in a Wheatstone bridge.

The balance type employed in this study is an internal force balance which consists of five forces and one moment: *NF1*, *NF2*, *SF1*, *SF2*, *RM* and *AF* **[1,3]**. *NF1* and *NF2* are combined to calculate the normal force and the pitching moment, *SF1* and *SF2* are combined to calculate the side force and the yawing moment. The units for bridge output, force and moment are volt, newton and newton×meter, respectively.

2. METHODS

2.1. *Internal balance calibration*

The calibration of an internal balance involves the application of known weights to the balance and recording strain-gages readings at each force and moment combination. A set of approximately fifty weights of nominal values 2.0, 1.0, 0.5 and 0.25 kg is used to apply the calibration loads. These weights are compared against standard weights of class F1, according to OIML terminology. In total, 81 loading combinations are employed. Loading number 1 is replicated eight times, i. e., loadings 1, 12, 21, 30, 41, 51, 60, 69 and 81 are similar. Table 1 presents some typical loading values.

Table 1. Examples of calibration loadings.

Aerodynamic Component	Loading number		
	3	17	58
AF (N)	0	0	39.2
SF (N)	0	-39.2	0
NF (N)	-78.4	0	0
RM (Nm)	0	0	7.64
PM (Nm)	0	11.76	0
YM (Nm)	-7.35	0	0

Calibration uncertainties in the weights applied for axial force are shown in Table 2. The first column identifies the numbers of the loadings where *AF* was charged. The second column discriminates the weights and the third is the combined uncertainty in the weights.

Table 2. Calibration uncertainties in the applied weights. $(\times 10^{-05}$ **kg)**

loading number	weights	uncertainty
8	27;37	5.76
9	37;34;27;32	7.55
19	37;30	5.29
20	37;34;30;27	6.89
28	34,35	4.54
29	34;35;38;30	6.88
31 to 35, 58	27;37;38;35	7.52
36 to 39	37;35	4.98
40	34,37	4.88
57	37;38	5.16
66	32;42	5.60
67	32;34;35;42	7.21
77	32;34	4.88
78	27;32;34;42	7.77

The internal balance is mounted inside a cross like structure, called the calibration body (Fig. 2).

The loading is performed by employing a system of trays, cables and pulleys (Fig. 3). There are 14 trays for the application of weights.

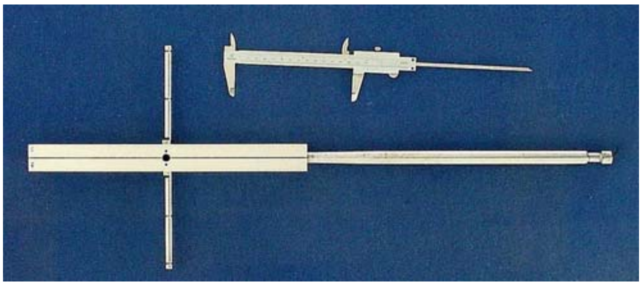


Fig. 2. Calibration body.

Fig. 3. Calibration system.

The nominal values of the weights and the identification of the trays used in the loading numbers 3, 17 and 58, chosen as examples, are presented in Table 3.

Table 3. Nominal Values of the weights applied to the calibration system.

Tray number	Mass (kg)		
	Loading 3	Loading 17	Loading 58
T1	0	0	2.0
T2	0	0	2.0
T3	0	0	0
T4	2.5	2.0	0
T5	2.5	0	0
T6	0	2.0	0
T7	0	0	0
T8	0	0	3.25
T9	0	4.0	0
T10	0	0	0
T11	2.0	0	3.25
T12	2.0	0	0
T13	2.0	0	0
T14	2.0	4.0	0

2.2. Functional relationship

The mathematical modeling of the calibration relates the aerodynamic components F to functions of the strain-gage bridges readings R (Eq. 1). The system is multivariate and consists of a linear combination of twenty seven functions of R [3]. These functions are called basis functions and correspond to: R_1, R_2, R_3, R_4, R_5, R_6, R_1R_1, R_1R_2, R_1R_3, R_1R_4, R_1R_5, R_1R_6, R_2R_2, R_2R_3,…, R_6R_6 . There are 27 adjustable parameters for each one of the six aerodynamic components. The model´s dependence on its parameters a and b is linear.

$$F_i = \sum_{j=1}^{6} a_{i,j} R_j + \sum_{j=1}^{6}\sum_{k=j}^{6} b_{i,j,k} R_j R_k \qquad (1)$$

As an example, for the axial force AF, Eq. (1) becomes:

$$\begin{aligned} AF = F_1 &= a_{1,1}R_1 + a_{1,2}R_2 + a_{1,3}R_3 + \ldots + a_{1,6}R_6 + \\ &+ b_{1,1,1}R_1R_1 + b_{1,1,2}R_1R_2 + \ldots + b_{1,6,6}R_6R_6 \end{aligned} \qquad (2)$$

Cross-product terms involving the input quantities are included in the polynomial because it is not possible to eliminate the interactions between bridges completely.

At each one of the 81 loadings, the bridge outputs are read several times and the mean values and standard deviations are computed.

2.3. Parameters estimation

According to the least squares methodology described in [4], a calibration curve is fitted to each set of the N = 81 data points (R, F). Each of the six aerodynamic components is arranged in a vector F whose dimension is 81×1. The design matrix R of the fitting problem has a dimension of 81×27 and is constructed from the basis functions.

Denoted by $\hat{p}$ the parameters of the fitting, the least squares results in:

$$\hat{p} = \left(R^T \cdot V^{-1} \cdot R\right)^{-1} \cdot \left(R^T \cdot V^{-1} \cdot F\right) \qquad (3)$$

R^T: transpose of the matrix R;
V: covariance matrix;
V^{-1}: matrix inverse of V.

Matrix $\left(R^T \cdot V^{-1} \cdot R\right)^{-1}$ is called the error matrix because it contains information about uncertainties of the estimated parameters. Its diagonal elements correspond to the variances (squared uncertainties $u_{\hat{p}}^2$) and the off-diagonal elements are the covariances of the fitted parameters, $u(\hat{p}_i, \hat{p}_j)$. It can be denoted by $V_{\hat{p}}$.

2.4. The covariance matrix

The covariance matrix V is related to the uncertainties of the applied loads u_F. It is made up of the contributions of the error sources due to the application of weights in the calibration system V_W and the uncertainties in the readings of the bridges V_R:

$$V = V_W + D \cdot V_R \cdot D^T \qquad (4)$$

2.5. Matrix V_W

In this study, V_W is considered an 81×81 diagonal matrix. Its elements are based on the uncertainties declared in the calibration certificates of the weights employed in the loading process and also in an estimation of the errors caused by the calibration system. The recognized system errors are misalignments of cables and pulleys and interactions between sensor bridges. A mathematical approach was employed to quantify the contribution of such errors, which consists of employing the fitting method described in section 2.3, but setting the covariance matrix equal to the identity in Eq. (3). This corresponds to first assigning an uncertainty value equal to 1 to the data points.

Following this, the uncertainty contribution to the set of measurements is approximated by the standard deviation of the fitting, S, which is the positive square root of the expression:

$$S^2 = \frac{1}{N-m}\sum_{i=1}^{N}\left(F_{i\,fitted} - F_{i\,applied}\right)^2 \qquad (5)$$

where:

N is the number of data points;
m is the number of parameters to be fitted;
$F_{i\ applied}$ is the applied load which corresponds to the weight applied to the calibration system; and
$F_{i\ fitted}$ is the fitted load value, evaluated through the least squares fitting.

2.6. Matrix $D \cdot V_R \cdot D^T$

The matrix $D \cdot V_R \cdot D^T$ corresponds to the variances and covariances in F due to uncertainties in the bridge readings and it is also 81×81. D is 81×6 and its elements are the sensitivity coefficients evaluated by taking the partial derivatives $\partial F/\partial R_j$ in Eq. (1). For the axial force, the 6 columns of matrix D are formed by:

$$\begin{aligned} \frac{\partial AF}{\partial R_1} &= a_{1,1} + 2b_{1,1,1}R_1 + b_{1,1,2}R_2 + b_{1,1,3}R_3 + \\ & b_{1,1,4}R_4 + b_{1,1,5}R_5 + b_{1,1,6}R_6 \\ \frac{\partial AF}{\partial R_2} &= a_{1,2} + b_{1,1,2}R_1 + 2b_{1,2,2}R_2 + b_{1,2,3}R_3 + \\ & b_{1,2,4}R_4 + b_{1,2,5}R_5 + b_{1,2,6}R_6 \\ & \ldots \\ \frac{\partial AF}{\partial R_6} &= a_{1,6} + b_{1,1,6}R_1 + b_{1,2,6}R_2 + b_{1,3,6}R_3 + \\ & b_{1,4,6}R_4 + b_{1,5,6}R_5 + 2b_{1,6,6}R_6 \end{aligned} \qquad (6)$$

Parameters a and b of Eqs. (6) were previously estimated by performing the least squares as shown in section 2.5.

The replication of loading number 1 supplies information for the construction of the 6×6 matrix V_R (Eq. 7), which represents the variances and covariances in the readings of the sensors. The output readings of the bridges resulting from the similar loadings 1, 12, 21, 30, 41, 51, 60, 69 and 81, are the basis for the calculation of the six standard deviations, u_{Ri}, and the covariances between readings, $u(R_i,R_j)$.

$$V_R = \begin{bmatrix} u^2_{R_1} & u(R_1,R_2) & \cdots & u(R_1,R_6) \\ u(R_2,R_1) & u^2_{R_2} & \cdots & u(R_2,R_6) \\ \vdots & \vdots & \ddots & \vdots \\ u(R_6,R_1) & u(R_6,R_2) & \cdots & u^2_{R_6} \end{bmatrix}_{6\times 6} \quad (7)$$

2.7. Goodness-of-fit

The goodness-of-fit is evaluated through the chi-square quantity, χ^2, defined as [5]:

$$\chi^2 = \sum_{i=1}^{81} \frac{\left(F_{i\,applied} - F_{i\,fitted}\right)^2}{u_i^2} \quad (8)$$

where u_i is the uncertainty in the applied load.

Taking into account the covariances, in matrix notation, Eq. (8) becomes:

$$\chi^2 = \left(F_{applied} - F_{fitted}\right)^T \cdot V^{-1} \cdot \left(F_{applied} - F_{fitted}\right) \quad (9)$$

A value for the chi-square which indicates a good fit is typically close to the number of degrees of freedom:

$$\nu = N - m \quad (10)$$

This leads to another quantity, the reduced chi-square:

$$\chi^2_\nu = \frac{\chi^2}{\nu} \quad (11)$$

whose desired value is approximately equal to 1.

2.8. The predicted load values

One can use the results of the internal balance calibration to predict the aerodynamic forces and moments which would act on a model tested in a wind tunnel. The predicted aerodynamic values correspond to the fitted loads F_{fitted} (Eq. 12). Choosing a particular set of readings, R_j, obtained in the calibration process, each aerodynamic component is estimated using its corresponding adjusted $\hat{p}$ parameters:

$$F_{fitted} = R_j \cdot \hat{p} \quad (12)$$

R_j is 1×27 and $\hat{p}$ is 27×1.

The uncertainty in the predicted aerodynamic component is the positive square root of:

$$V_{F_{fitted}} = R_j \cdot V_{\hat{p}} \cdot R_j^T \quad (13)$$

Equation (13) is equivalent to applying the law of propagation of uncertainty in Eq. (1)[2].

3. RESULTS AND DISCUSSION

The least squares regression provides the fitted parameters vector $\hat{p}$, its variances and covariances. The calibration data reduction also includes the predicted load values and their uncertainties. The quality of the fit is quantified by the chi-square.

Code in MATLAB® and Excel® worksheets were used to perform the data reduction.

To check for numerical instabilities in matrix inversion, determination of the rank and QR decomposition of matrix V was performed. The rank estimate revealed that rows and columns of V are linearly independent **[6]**.

3.1 Covariance matrix

The uncertainties declared in the calibration certificates of the weights are combined with an estimation of the uncertainty due to the error caused by the loading system. The difficulty of experimentally addressing this estimation led to the option of statistically quantifying it. So, the procedure presented in section 2.5 was employed to supply the standard deviation S of the fitting.

Table 4 presents the standard deviations of the six aerodynamic components. The axial force and rolling moment standard deviations were the lowest estimated values and were chosen as an indication of the error associated to the calibration system.

The better behavior of these components was expected as individual bridges were employed to measure these components, whilst the results of the other components were taken from a combination of bridge readings.

The reason for choosing the lowest values is a question of metrological goal in achieving the best level of repeatability as possible in the calibration process.

For the three force components, contributions assigned to the diagonal matrix V_W, due to the calibration system, are around 1.5×10^{-2} squared. For the three moment components, this value is 6.6×10^{-2} squared.

This choice is arbitrary and is based on 10 % of the standard deviations S, i.e., it is assumed that the system contributes only this amount to the dispersion of the data.

Table 4. Standard deviation. Unit: N (force), Nm (moment of force).

load	*AF*	*SF*	*NF*	*RM*	*PM*	*YM*
S	0.153	0.590	0.403	0.066	0.073	0.125

Bridge readings for loadings described in Table 5 are the basis for the covariance matrix V_R (Table 6).

The matrix is diagonally symmetric. Main diagonal elements are variances. Off-diagonal elements represent covariances.

Table 5. Replication for loading number 1. Unit: mV .

loading	Bridges output readings					
	R_1	R_2	R_3	R_4	R_5	R_6
1	0.114	-0.085	5.596	-0.170	-0.625	0.127
12	0.107	-0.076	5.605	-0.186	-0.625	0.134
21	0.102	-0.111	5.598	-0.171	-0.622	0.125
30	0.090	0.143	5.617	-0.220	-0.627	0.130
41	0.114	-0.070	5.602	-0.175	-0.622	0.125
51	0.114	-0.057	5.606	-0.187	-0.622	0.128
60	0.119	-0.145	5.597	-0.184	-0.620	0.127
69	0.083	0.221	5.606	-0.248	-0.625	0.132
81	0.116	-0.098	5.596	-0.168	-0.621	0.123

Table 6. Matrix V_R. All elements are multiplied by 1×10^3.

0,16	-1,28	-0,05	0,26	0,02	-0,02
-1,28	15,55	0,55	-2,82	-0,20	0,21
-0,05	0,55	0,05	-0,11	-0,01	0,01
0,26	-2,82	-0,11	0,72	0,03	-0,06
0,02	-0,20	-0,01	0,03	0,01	0,00
-0,02	0,21	0,01	-0,06	0,00	0,01

3.2 Parameters estimation

Performing the least squares fitting considering uncertainties in the data points (Eq. 3), results in the parameters presented in Table 7.

Table 7. Estimated parameters.

	AF	*SF*	*NF*	*RM*	*PM*	*YM*
p_1	-22.048	-0.394	-0.590	-0.031	-0.017	0.017
p_2	-2.392	-6.448	0.187	-0.011	0.005	-1.887
p_3	0.135	-1.214	-1.337	-0.083	-2.135	-0.014
p_4	-0.132	0.511	0.248	-2.380	0.024	0.012
p_5	0.456	0.532	51.079	0.035	-0.284	0.051
p_6	3.272	46.033	-0.228	-0.013	0.013	-0.085
p_7	0.095	-0.116	0.005	-0.007	-0.003	0.006
p_8	0.002	-0.058	0.155	-0.001	0.048	0.008
p_9	0.002	-0.028	0.024	-0.004	0.000	0.007
p_{10}	0.007	-0.002	0.000	0.022	0.003	0.006
p_{11}	-0.008	-0.114	-0.156	-0.002	0.013	0.007
p_{12}	-0.073	-0.025	-1.075	0.001	0.003	0.001
p_{13}	-0.034	0.029	0.004	-0.002	0.015	0.010
p_{14}	0.015	0.013	-0.031	0.002	0.000	0.004
p_{15}	-0.014	-0.007	-0.104	0.005	-0.040	0.001
p_{16}	0.009	-0.023	-0.032	0.003	-0.002	-0.006
p_{17}	-0.478	-0.224	0.135	0.014	-0.092	-0.061
p_{18}	0.020	0.024	0.005	0.001	0.001	0.002
p_{19}	0.219	-0.045	-0.019	-0.001	0.000	0.030
p_{20}	-0.213	-0.217	-0.011	-0.002	-0.001	-0.002
p_{21}	-0.013	-0.020	0.009	0.003	0.002	-0.010
p_{22}	-0.526	-0.055	-0.011	-0.004	0.002	0.001
p_{23}	0.128	-1.046	-0.006	0.000	0.000	0.044
p_{24}	-0.010	-0.032	0.835	-0.003	-0.008	0.004
p_{25}	0.687	-0.192	0.019	0.031	0.008	0.006
p_{26}	0.010	0.125	0.078	-0.011	-0.003	0.013
p_{27}	0.158	0.099	0.712	-0.007	-0.074	0.010

The values of the applied and fitted aerodynamic forces and moments, along with the corresponding uncertainties are presented in Table 8, for the loading numbers 3, 17 and 58. Units: newton (force) and newton×meter (moment).

The uncertainties associated with the aerodynamic forces (*AF*, *SF*, *NF*), as well as those ones associated with the aerodynamic moments (*RM*, *PM*, *YM*) are similar to each other, as expected. This fact results from the dependence of the values used in the loadings and mainly, from the uncertainty values attributed to the system.

Table 8. Applied, predicted and uncertainty of aerodynamic forces and moments.

load component	Loading number 3	17	58
$AF_{applied}$	0	0	39.2
AF_{fitted}	0.01	0.03	40.77
u_{AF}	0.11	0.11	0.12
$SF_{applied}$	0	-39.2	0
SF_{fitted}	0.26	-38.96	0.09
u_{SF}	0.11	0.11	0.12
$NF_{applied}$	-78.40	0	0
NF_{fitted}	-69.08	-0.75	0.01
u_{NF}	0.11	0.11	0.12
$RM_{applied}$	0	0	7.64
RM_{fitted}	-0.01	0.01	6.44
u_{RM}	0.05	0.05	0.05
$PM_{applied}$	0	11.76	0
PM_{fitted}	-0.06	11.73	0.04
u_{PM}	0.05	0.05	0.05
$YM_{applied}$	-7.35	0	0
YM_{fitted}	-7.32	-0.07	1.41
u_{YM}	0.05	0.05	0.05

3.3 Goodness of fit

The chi-squared χ^2 and reduced chi-square χ^2_ν values of the fitting are presented in Table 8.

Table 9. Goodness of fit.

Load component	χ^2	χ^2_ν
AF	56.23	1.04
SF	835.77	15.48
NF	390.14	7.22
RM	56.22	1.04
PM	70.03	1.30
YM	203.66	3.77

Estimating the chi-square should result in a value around the number of degrees of freedom of the polynomial fit **[4]**. As there are 81 equations and 27 unknowns, the number of degrees of freedom is equal to 54. The magnitudes presented in Table 9 are greater than expected, revealing that either the math model or the quantification of the uncertainties presented in the experiment, or even both, should be better investigated. Also, the high value of the second element of the main diagonal of matrix V_R reveals that something might not have behaved properly during the calibration. Problems when fixing the internal balance inside the calibration body could have been the cause of the high dispersion of the bridges readings.

After improving the configuration of the calibration system, studies including short, medium and long term calibration data should be performed.

4. CONCLUSIONS

A methodology for the estimation of uncertainty in internal balance calibration was proposed. Results of the curve fitting include the estimation of the values and uncertainties of the fitting parameters, as well as a statistical measure of goodness-of-fit. The law of propagation of uncertainty was employed to estimate the uncertainties in the aerodynamic components.

The identification of error sources caused by the calibration system and their consequent contributions to the uncertainty of the load components has still not been completely accomplished. A previous value for this contribution was mathematically derived assuming that all measurements had the same standard deviation and a new set of fitting parameters was recomputed. This approach makes the method iterative, which implies that an independent procedure for the assessment of goodness-of-fit must be conducted. A test of a standard aeronautical model, such as the AGARD model, employing the calibrated internal balance, may be a way to compare the aerodynamic loads evaluated by the calibrated data.

The uncertainties declared in the calibration certificates of the weights and the dispersion of the readings of the bridges are the only contributions experimentally estimated. They are not dominants.

Other expected influences as misalignment of the system, interaction between bridges, and hysteresis effects are still to be quantified.

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