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ON SOME KEY CONCEPTS AND TERMS IN MEASUREMENT HAVING A CROSS–DISCIPLINARY IMPACT

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Abstract: A discussion about some key concepts and terms in measurement is proposed for promoting a fruitful dialogue among scientists of different disciplines having an experimental basis. This revision of terms requires both the examination of their current use in the involved communities and an investigation on their role in scientific theories. In the present paper, this second aspect is mainly pursued and key terms such as (measure) value, measuring system, measurement value and measurement model are discussed. Two theories of measurement are considered, deterministic and probabilistic, the former is apt to describe an ideal measurement process, the latter is able to account for uncertainty. The meaning of the measure value in a probabilistic contest is elicited and a new taxonomy of uncertainty sources is proposed. Lastly, the role of modelling in measurement is discussed.

Keywords: measurement science, measurement of quantities related to human perception, probabilistic theory of measurement.

1. INTRODUCTION

For many years the clarification of terminology has been a major concern in metrology and measurement and has been pursued by the development of the International vocabulary of metrology (VIM) [1-3], now at its third edition as well as by intensive research about the foundation of measurement [4-5]. The motivation behind the first and, even more, the second revision of the VIM has been to “take account of the needs of chemistry and related fields” [2] and to “cover measurements in chemistry and laboratory medicine for the first time” [3], since “it is taken from granted that there is no fundamental difference in the basic principles of measurement in physics, chemistry, laboratory medicine, biology and engineering”. Therefore there is a trend to increase the number of disciplines that may benefit from a reorganisation of measurement knowledge. In this regard an even more ambitious perspective may be envisaged. Recently the European Community has paid close attention to the problem of the measurement of quntities related to human perception, issuing a Call named Measuring the Impossible, in the NEST (New and Emerging Science and Technology) programme of the 6th Framework Programme [6]. In this context a Coordination Action, MINET (Measuring the Impossible Network) [7] has been launched aiming to promote a close dialogue among different discipline scientists working in this field. In the so far developed coordination activities, including workshops and “think tank” events [8], the need of finding a common language has been soon raised. We think that this is a new and challenging frontier worthy of serious investigation.

It is necessary to consider the current understanding of basic concepts and the current use of basic terms in the involved communities, as well as the links between terms and theories. In the present paper, we pursue particularly this second aspect. After a brief introductory discussion, we concentrate on four key concepts and terms, (measure) value, measuring system, measurement value and measurement model, and we discuss them with reference to two measurement theories, deterministic and probabilistic.

2. SOME BASIC CONCEPTS AND TERMS

Let us try to identify *a minimum set of terms virtually necessary for any discourse about measurement*.

We first need a term for denoting what we (want to) measure. We could call this a property, characteristic, attribute, feature of something. Let us choose, for example, *characteristic*. Then we have to name what carries (expresses, manifests) the characteristic under investigation: it may be an object, an event or even a person. In this case the differences are substantial, yet we will still use a unique term, *object*, but making clear that this is just a conventional term for denoting what carries the property. Then we have to distinguish between *measurable* and *non measurable* characteristics: for the former the term *quantity* may be used, although this term has somehow a stronger meaning, since it is understood to imply an underlying metric scale. Furthermore, since objects manifest the characteristic of interest in different ways (levels, degrees), we will call *state* the way in which an object manifests a characteristic.

Now we are ready for introducing measurement. Yet we do not want to discuss here a definition of measurement, but rather some key concepts and terms related to it. A summary of the main terms considered in this paper is shown in Table 1.

Table 1. General terms for measurement.

Object	the carrier of the characteristic to be measured; it may be a physical object, an event or a person
Characteristic or property (of an object)	what we want to measure
Measurable characteristic (or quantity)	a characteristic that may be measured
State (of an object, with respect to a given characteristic)	particular way in which an object manifests a characteristic
Empirical relation	a relation that may be observed between two or more objects, with reference to the characteristic of interest
Empirical structure (or empirical relational system)	a set of objects and a set of empirical relations on it
Nominal, difference, interval, extensive	different types of empirical structures
Numerical structure (or numerical relational system)	a set of numbers and a set of numerical relations on it
Scale	(general meaning) the set of formal conditions for measurement (an empirical structure, a numerical structure and a measure function constituting an homomorphism between them)
(Reference) scale	(specific meaning) a series of standard objects with corresponding numerical values properly assigned
Nominal, ordinal, interval, ratio	different types of scales
Measuring system (or instrument)	an empirical system able to interact with objects carrying the characteristic under investigation and to produce, as a result of the interaction, an observable output, on the basis of which it is possible to assign a value to the object to be measured
Measurement process	the process through which a value is assigned to a measurand, normally based on the use of a measuring system
Calibration	the operation through which the characteristics of the measuring system are assessed
Model	an abstract systems that represents, to some extent and from some standpoint, a real system (or a class of real systems)
(Natural) law	a functional relation (or a model) that links one or more characteristics of real objects
Measurement model	a model assumed for founding the measurability of some characteristic or for performing some measurement

3. (MEASURE) VALUE

Measurement concerns the description of objects through numbers. We will call *(measure) values* such numbers. But how do they describe objects (with respect to the characteristic under investigation)? The most accredited solution comes from the representational theory, according to which measure values describe the objects in that they reproduce, in a numerical domain, the empirical relations holding among them [4, 9]. This is called a representation. Yet we may consider either a deterministic or a probabilistic representation and the interpretation of the measure value is different in the two cases. Let us see what happens.

3.1. Deterministic representation

The representational framework could be summarised as follows. Consider a set of objects *A* showing the characteristic *x*. Assume that some classes of empirical relations can be observed about those objects with respect to *x*: if they satisfy some properties, it is possible to assign numbers, that we call *measure values*, to the objects in such a way that the relations among those numbers reproduce the empirical relations. This assignment is in general not unique since there are admissible transformations maintaining the same representational properties. Yet, suppose that we fix by proper conventions the degrees of freedom and then we make the assignment strictly unique. This assumption simplifies the discussion and does not reduce the discourse generality. Therefore, for any object *a*, in the set *A*, *the measure value is* ***that*** *value fulfilling all the constraints resulting from the empirical relations* (plus the additional conventional constraints)*, once that they have been mapped into the corresponding numerical ones.* For example, consider the set $A=\{a,b,c\}$ and suppose that the following empirical relations hold: $a \succ b \sim c$. If we map them in a numerical domain, we obtain the conditions $m(a)>m(b)=m(c)$, i.e., a system of inequalities, which are satisfied by the following measure values: $m(a)=2$, $m(b)=1$ and $m(c)=1$. The representation for ordinal, interval and ratio scales is summarised in Table 2.

This framework is deterministic and consequently it provides an idealised (oversimplified) picture of the empirical reality. For example this description implies that for any couple of objects, say *a* and *b*, a definite and perfectly reproducible relation always holds. Let us then see how this limitation may be overcome.

Table 2. Synopsis of the theory [10] – deterministic versus probabilistic approach.

The measurement scale		
Scale type	*Deterministic approach*	*Probabilistic approach*
Order	$a \succcurlyeq b \Leftrightarrow m(a) \geq m(b)$	$\mathbb{P}(a \succcurlyeq b) = P(x_a \geq x_b)$
Interval	$\Delta_{ab} \succcurlyeq \Delta_{cd} \Leftrightarrow$ $m(a)-m(b) \geq m(c)-m(d)$	$\mathbb{P}(\Delta_{ab} \succcurlyeq \Delta_{cd}) =$ $= P(x_b - x_a \geq x_d - x_c)$
Ratio	$a \sim b \circ c \Leftrightarrow m(a) = m(b) + m(c)$	$\mathbb{P}(a \sim b \circ c) = P(x_a = x_b + x_c)$
The measurement process		
Process	*Deterministic approach*	*Probabilistic approach*
Observation	$y = f(x)$	$P(y \mid x)$
Restitution	$\hat{x} = f^{-1}(y)$	$P(x \mid y) = P(y \mid x)\left[\sum_{x \in X} P(y \mid x)\right]^{-1}; \hat{x} = \mu(x \mid y)$
Measurement	$\hat{x} = g(x) = x$	$P(\hat{x} \mid x) = \sum_{y \in Y} \delta[\hat{x} - \mu(x \mid y)] P(y \mid x)$

3.2. Probabilistic representation

A more realistic description of empirical relations may be obtained by a probabilistic representation [10]. Consider again the simple example above and suppose now that element *a* is fairy "distant" from *b* and *c*, so that whenever we compare it with them we observe that it is greater. We may express this by assigning $\mathbb{P}(a \succ b) = \mathbb{P}(a \succ c) = 1$, where $\mathbb{P}$ is a probability function defined over empirical relations. On the other hand, suppose that *b* and *c* are very close to each other, so that when we compare them, sometimes we observe that they are equivalent, some other times that they differ[1]. For example, it may happen that $\mathbb{P}(b \succ c) = \mathbb{P}(c \succ b) = 0.1$ and $P(a \sim b) = 0.8$ [2]. This is basically how a probabilistic framework works [11]. Note the difference with the deterministic one: in that case we have only one possible order[3], $a \succ b \sim c$, whilst in this we have three of them,

1. $\succcurlyeq_1 : a \succ b \sim c$,
2. $\succcurlyeq_2 : a \succ b \succ c$,
3. $\succcurlyeq_3 : a \succ c \succ b$,

where we have introduced the notation $\succcurlyeq_i$ for denoting the *i*–th order. It is easy to verify that these orders, although all possible, have different probabilities, namely

1. $P(\succcurlyeq_1) = 0.8$,
2. $P(\succcurlyeq_2) = 0.1$,
3. $P(\succcurlyeq_3) = 0.1$.

But what about the measure value in this context? As we have several different possible empirical orders, we have also different possible value assignments. It is apparent that, corresponding with each empirical order, the following assignments are appropriate:

1. $\succcurlyeq_1 \Rightarrow m(a) = 2$, $m(b) = 1$, $m(c) = 1$,
2. $\succcurlyeq_2 \Rightarrow m(a) = 3$, $m(b) = 2$, $m(c) = 1$,
3. $\succcurlyeq_3 \Rightarrow m(a) = 3$, $m(b) = 1$, $m(c) = 2$.

Then a "natural" mathematical approach consists in describing each object by a *random variable* accounting for the different numerical values that may be associated to the object. Let us denote by x_a, x_b and x_c such random variables. As result their probability distributions will be as follows:

1. $P(x_a = 3) = 0.2$,
   $P(x_a = 2) = 0.8$, $P(x_a = 1) = 0.0$,
2. $P(x_b = 3) = 0.0$,
   $P(x_b = 2) = 0.1$, $P(x_b = 1) = 0.9$,
3. $P(x_c = 3) = 0.0$,
   $P(x_c = 2) = 0.1$, $P(x_c = 1) = 0.9$.

The notion of measure value still holds, but with a fairy different interpretation. Now *the measure value is a random variable associated to each object, whose values are the values satisfying the possible empirical structures*; *each value has a probability related to the probabilities of the empirical structures*[4]. Incidentally, note that the different values that each random variable may assume can be interpreted as representations of the possible states of the object (with respect to the characteristic under investigation). As result, whilst in the deterministic framework each object is uniquely associated to a state, which in turn is uniquely represented by a number, in this case each object can exhibit different states, each of them uniquely represented by a number. A probabilistic representation for ordinal, interval and ratio scale is summarised in Table 2, along with a comparison with the deterministic one.

[1] Incidentally, we may note that generally at this level it will not be possible to decide whether this variability is due to the comparator or if it is inherent to the objects.

[2] Note that, of course, it must be $P(b \sim c) + P(b \succ c) + P(c \succ b) = 1$ Note also that in this paper we are not interested in how we can actually evaluate the probabilities involved, but only in their meaning, in what they express.

[3] In general we would say only one possible empirical relational structure.

[4] Precisely, the probability of each value equals the sum of the probabilities of the empirical structures in which that value is appropriate.

In conclusion, we can say that the notion of measure value is essential for a measurement theory and that in a probabilistic theory the measure value becomes a random variable, which is able of properly accounting for the uncertainty, which is inherent to the empirical relations founding the measurability of the characteristic under consideration. Note some consequences:

1. this representation accounts for the uncertainty of primary standards; it may be particularly useful for "extreme" scales of great current metrological interest, such as the "nanoscale" [12];
2. it provides a theory for the measurement of quantities related to human perception, much needed, as we have discussed in the introductory section [13];
3. it accounts for the intrinsic (or definition) uncertainty, as we will discuss later on [3, 14].

A distinction between the measure value and the measurement value is now in order. This is where the notion of measuring system comes into play.

4. THE MEASURING SYSTEM

So far we have discussed the meaning of the measure value. Let us now discuss how it may be assigned to a real object. In principle, we could proceed as follows.

Suppose that A is finite, with N elements and $n < N$ possible states. We examine all the objects and we test all the relations (of interest) among them and we assign the measure values according to them. This is what is actually done, sometimes, in psychological or sociological scaling, when a limited number of objects are considered and the results are intended to hold basically for the set under test. The limitation of this method is evident: as soon as the number of objects increases it becomes inapplicable. Yet this method remains essential for *starting*, at least in principle, any measurement process. In general, we can say that any measurement process requires the previous *construction of a reference scale*. This process involves, at least in principle, the following steps:

- select object samples, one for each of the n possible states, and form with them a series of standards $S = \{s_i \mid i = 1,...,n\}$,
- assign a measure to each of them following the procedure described above and
- form a reference scale, that is the association of each standard with its measure value $R = \{(s_i, m(s_i)) \mid i = 1,...,n\}$.

Once that the reference scale is available, the measurement of the object a, not included in the reference scale, may be done by comparing it with the scale, which in turn may be done either directly or indirectly by a calibrated measuring system [15]. In both cases we will call measuring system (MS) the device that we use for performing the measurement, once that the reference scale is given.

Thus, we propose to define the MS as *an empirical system that is able to interact with objects carrying the quantity under investigation and to produce, as result of the interaction, an observable output, the (instrument) indication, on the basis of which it is possible to assign a value to the object to measure*. Therefore we can distinguish two phases in the measurement process (MP) ,

- the production of indications by the MS and
- their interpretation yielding the assignment of measurement values to the objects.

We call *observation* the former, *restitution* the latter [16]. We need to distinguish the two phases: observation is in general a chain of physical transformations, possibly including perception and judgement, if we consider the psychological measurement as well, whilst observation is a data processing phase. We make a distinction also between measuring system and measuring process: a measurement process is based on the use of a measuring system but it includes the measuring system and the way it is used, the measurement procedure. The restitution may be embedded in the measuring system or it may be implemented by a post–processing of the instrument indications. Anyway, since the MP is strongly characterised by the MS, sometimes we will use the two terms quite freely. Note also that we have introduced the term measurement value to denote the MP final output and we have distinguished it from the measure value discussed earlier. In the ideal model, as we will see in a moment, these two values coincide, whilst generally in the probabilistic model they differ, as a consequence of the uncertainty sources associated to the measuring system. Let us now see how the MS and the MP may be modelled, first in a deterministic and then in a probabilistic way.

4.1. Deterministic model of the measuring system and of the measurement process

Observation may be simply modelled as a mapping, φ, from the set of objects, A, into the set of indications, Y, that is $\varphi : A \mapsto Y$. Now a major question arises: what is the property actually characterising the MS?

The answer is relatively simple, since the MS has to be sensitive to the object state, but not to its "identity", in other words, different objects in the same state, for example having the same length, mass… must give rise to the same response. This is reasonable and also easy to state: it must be that, for each a, b in A,

$$a \sim b \leftrightarrow \varphi(a) = \varphi(b). \tag{1}$$

Yet there is another, more usual, way of characterising the measuring system: we may consider the way it responds to the states of the input object, or, equivalently, to their (measure) values, since states and values are in a one to one correspondence. Let then X be the set of the measure value, then we may introduce another function, f, which we call the MS characteristic function, $f : X \mapsto Y$, linked to φ by the equality

$$\varphi(a) = f[m(a)]. \tag{2}$$

After observing the indication y, we must be able to identify the state/value of the object, which caused it. This is possible if the function f is invertible. If we denote by $\hat{x} \in \hat{X}$ the measurement value, restitution is simply described by a mapping, $g: Y \mapsto \hat{X}$, such that

$$\hat{x} = g(y) = f^{-1}(y). \tag{3}$$

Finally, the whole measurement process is described by the mapping $h: X \mapsto \hat{X}$, resulting from the concatenation of observation and restitution, that is

$$\hat{x} = h(x) = g[f(x)] = f^{-1}[f(x)] = x, \tag{4}$$

which, in the ideal case, results into a unitary transformation. Note anyway, that although the measurement transformation in the ideal case is unitary, it is essential to consider it, since the measure value x is an unobservable variable (if we would be able to directly observe the value of the measurand we would not need to do any measurement!) whilst $\hat{x}$ is an observable variable, since it results from a numerical processing of the observable indication y.
The deterministic model of the measurement process is summarised in Figure 1.

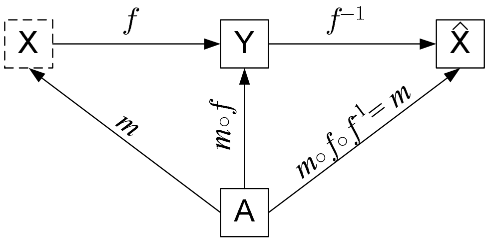

Fig. 1. Deterministic framework for measurement

4.2. Probabilistic model of the measurement process

The deterministic model paves the way to the probabilistic one, which is more realistic. In this perspective, observation is no more describable by a deterministic function, since for each object under test we will no more obtain a unique indication, y, but rather a plurality of possible indications, ruled by a probability distribution. Instead of the function $y = \varphi(a)$, now we have a parametrical random variable y_a, characterised by a conditional probability distribution $P(y \mid a)$.
Yet we still require that the MS responds only to the object state, and is not affected by the object identity. This may be stated now by requiring that, for each $a, b \in A$,

$$P(y \mid x_a = x) = P(y \mid x_b = x). \tag{5}$$

Observation may be also described by a parametrical random variable y_x, characterised by a conditional probability distribution $P(y \mid x)$, replacing the characteristic function $y = f(x)$. In our notation, P denotes a probability distribution for a random variable and its argument specifies the variable it refers to.

Now f can be re–interpreted as $f(x) = \mu(y_x)$, where μ is a proper position parameter for y_x (as the expected value, if appropriate), which is consistent with the way in which an instrument characteristic function is usually understood.

In analogy with the deterministic case, restitution is now the probabilistic inversion of the observation transformation, which can be obtained with the *Bayes–Laplace* rule, as

$$P(x \mid y) \propto P(y \mid x), \tag{6}$$

if we assume a uniform prior distribution for x, which is usually appropriate in this context. The measurement value can be defined as

$$\hat{x} = \mu(x \mid y),$$

which is still a deterministic function of the indication y, and we can derive any required uncertainty figure from the distribution (6).
Lastly, the overall measurement process can be described by the concatenation of observation and restitution, yielding a parametrical random variable $\hat{x}_x$, characterised by

$$P(\hat{x} \mid x) = \sum_y \delta[\hat{x} - \mu(x \mid y)] P(y \mid x), \tag{7}$$

where δ is a discrete Dirac operator. It is reasonable to expect, although we do not have a formal proof for this, that the probabilistic mapping characterising the overall measurement process is still unitary in the average, that is

$$\mu(\hat{x} \mid x) = x, \tag{8}$$

which corresponds to assert that the measurement process is unbiased.

Note the difference between (6) and (7) [21]:

- the former is appropriate for providing the final result, with a proper uncertainty statement, whenever a specific measurement has been done and the indication *y actually* acquired,
- the latter is appropriate for declaring (or evaluating) the performance of a measuring process for any *hypothetical* value of the measurand *x*.

The probabilistic model in its various branches is depicted in Figure 2.

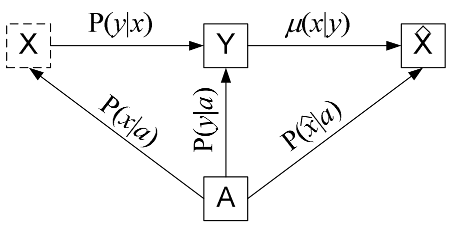

Fig. 1. Probabilistic framework for measurement

In this probabilistic framework both the measure value and the measurement value are represented as random variables: this is a key point worthy of a thorough discussion.

5. MEASUREMENT VALUE AND UNCERTAINTY SOURCES

We have seen that in the probabilistic framework the measure value itself becomes a random variable accounting from the uncertainty related to the empirical relations, which are no more considered as perfectly determinable. In particular this will be the uncertainty to be assigned to the elements of the reference scale. This complies with the common assumption that the uncertainty of the standards forming the reference scale is the minimum uncertainty achievable for the characteristic under investigation. By extension, we can assume as well that the uncertainty related to the measure value may be viewed as "intrinsic" to the characteristic under investigation: it seems reasonable to assume that *we would attain at minimum uncertainty if we were able to directly compare all the objects among them*, provided that, if we have different ways of making this comparison we choose the most accurate. Yet, what we obtain at the end of the measurement process is not the measure, but rather the measurement value. This value is again a random variable and it is very reasonable to assume that its uncertainty will be in general greater than the measure value, since it will include also a quota due to the measurement process. This is what we want to discuss now. We have seen that the MS may be characterised by the distribution $P(y \mid x)$. The point is that this distribution may be obtained, at least in principle, by calibration. Let us then discuss calibration. Calibration is based on the application of a standards series S to the measuring system and the observation, recording and processing of the corresponding observations y. Then, in principle, the result that we obtain is a probability distribution $P(y \mid s)$, whilst our goal is $P(y \mid x)$: in fact *we do not observe directly the input values, x, but rather the standards s.* How can we obtain the $P(y \mid x)$? We should remember that we have already established the reference scale and as consequent we have assigned to each standard s the random variable x_S, or, equivalently, we know the distribution $P(x \mid s)$. Using again the *Bayes–Laplace* rule, we may invert it and obtain

$$P(s \mid x) \propto P(x \mid s), \tag{9}$$

from which we calculate

$$P(y \mid x) = \sum_s P(y \mid s) P(s \mid x). \tag{10}$$

Note now that the distribution $P(y \mid x)$ includes two uncertainty contributions,

- the uncertainty due to the transformation performed by the MS,
- an additional uncertainty quota due to the fact that we cannot directly observe the input for the calibration.

The latter is basically the uncertainty of the reference scale, that is the minimum uncertainty that we can have for the characteristic under consideration. Moreover, in the measurement process we can have two main kinds of uncertainty sources, related respectively to random variations and systematic effects. We have already seen that the former may be described by the distribution $P(y \mid x)$. The systematic effect due to an influence parameter, say θ, may be modelled by considering the distribution $P(y \mid x, \theta)$, conditioned also by θ. In this case restitution will be provided by

$$P(x \mid y) = \sum_\theta P(x \mid y, \theta) P(\theta), \tag{11}$$

where

$$P(x \mid y, \theta) \propto P(y \mid x, \theta). \tag{12}$$

When we measure the object a, we provide the result through the distribution $P(x \mid y)$. Note now the meaning of this result: this is the appropriate description if we assume that *the object, in the moment (or in the time interval) in which we perform the measurement can be described by a constant value or, in other terms, is in a constant well defined state*. This description is not the same as that provided by the random variable $\hat{x}_a$, which also accounts for the *possible indetermination of the state/value of the object*. Properly we have

$$P(\hat{x} \mid a) = \sum_x P(\hat{x} \mid x) P(x \mid a). \tag{14}$$

To summarise, in the association of a measurement value to an object, we have, in the most general case, four fundamental sources of uncertainty:

1. the object intrinsic uncertainty,
2. the uncertainty due to the measurement transformation, both resulting from
    a. random variations and to
    b. systematic effects,
3. the uncertainty related to the standards used for calibration.

6. MEASUREMENT MODEL

So far we have often used the term "model" and it could be useful to briefly discuss it. We think that scientific and technical knowledge heavily relies on models: perhaps we could even say that in the scientific investigation we always propose models and check if they fit or not with some aspects of the empirical reality. This is the meaning of Galileo's "sensible experiences": we design experiments basing ourselves on assumptions, which are models, either embryonic or fully developed in general theories, and then we check whether they verify or falsify the assumptions. Tentatively, we can say that *a model is an abstract system intended to represent, to some extent and from some standpoint, a real system (or a class of real systems).* The relationship between a model and a theory is also of import. We will not make a sharp distinction between the two, rather we will consider a theory as a very general model. Depending on the discourse level, we will decide what to consider a theory and what a model. In some cases, the model may be regarded as a special instance of a theory.

Let us now consider a key question: how can we check whether a proposed model actually represents the part of reality it intends to represent? This is the time where the measurement comes into play: the very task of measurement is to check the agreement between models and reality, i.e., between abstract (mental) systems and empirical systems.[5] But there is another side of the problem, particularly relevant here. Measurement, as a scientific activity, is subjected to being modelled itself. This leads us to the concept of *measurement model*, that we define as *a model assumed for founding the measurability of a characteristic or for performing a measurement*. Three cases may be envisaged.

6.1. Modelling for measurability

Concerning measurability [19], we can distinguish two levels, either measurability in the large or local measurability. For the former, we intend the problem of proving the measurability of a characteristic in general (for example whether hardness is measurable or not) or to select the best way of constructing a reference scale (the best way for implementing the metre falls in this category).

For local measurability we intend the problem related to measuring a special class of objects (for example a class of workpieces, such as mechanical pivots). In general, for both levels we can consider three types of models.

1. Models based on the internal properties of a characteristic

To found the measurability of length we can use the model of "length of a segment" if we are considering a class of rods, or, more generally, objects that are similar to segments, whilst we may use the model of "distance between parallel planes", if we deal with objects like blocks, and so on. It would be not easy to define length in general. A good question is: in which way the above models guarantee the measurability of length? They do that since in the considered models the property of order and the operation of addition are defined: as far as the model fits well the empirical reality, the real objects (rods) will show, *to some extent*, similar properties to the reference abstracts ones (segments). On the other hand there may be cases, for example in the field of perception, in which we do not have a well established model guaranteeing the measurability, rather we may prove it by empirically assessing that required properties are satisfied. Note that in this last case we are using the representational framework itself as a model: we may perhaps say that, at least in this case, the representational framework is the most general model to found measurability. In the following we will turn this remark into a general claim.[6]

2. Models dealing with influence quantities

In a mature domain of knowledge, the characteristics under consideration are not isolated, but rather they form a system and they are linked by relations (sets of equations) accepted in the system. This is the case for the international system of metrology. On the other hand, a real object is usually characterised by a set of quantities, linked by equations. Even if one is interested in only one of them, the others must be also accounted for, since they can affect the measurement. Then, usually the model used for founding measurability and designing measurement in the local context includes some quantities, one of which is *the measurand*, the others the *influence quantities*. The model "founds" measurability in the sense that it allows evaluating whether the effect of influence quantities is acceptable for the target uncertainty. If it is not, the model may lead the design of corrections. Suppose for example that we are considering length, l, and we want to account for the influence of temperature, t, as well. Consider two rods, a, b, and suppose that if they are at the same temperature, a has a greater length than b, that is $a \succ b$. Suppose now that temperature may be different from one object to another. Then we must evaluate with the model whether it may happen that due to temperature variations we could observe $b \succ a$. The model will enable us to prevent this from happening and/or to evaluate the uncertainty related to this thermal effect. The final question is anyway reducible to deciding on a representational framework.

3. Models for derived measurement

The most direct way of assessing measurability is by checking the existence of appropriate internal properties for the characteristic under consideration. A remarkable alternative is the use of "natural laws". For example, a physical quantity such as density, ρ, may be measured indirectly by measuring mass, m, and volume, V, and using the relation $\rho = m/V$. A psychophysical quantity such as loudness, L, may be measured indirectly by first measuring the related physical intensity, I, and then applying an accepted psychophysical law, such us Stevens law $L = \alpha I^{\beta}$. It seems that we have eluded the representational framework, yet there are arguments for claiming that it still applies: see Ref. [19] for a discussion.

Finally, we can perhaps summarise this issue, *modelling for measurement*, by saying that *a class of measurement models concerns the measurand and could be seen as related to measurability. The most general of these models is the representational framework itself.*

6.2. Modelling for measurement

The second instance in which we use a measurement model is for performing measurement. In this paper we do not have to consider only the measurand object but also the measuring system and the interaction between the two. We have already seen that virtually the most general model for the measurement process can be formulated by the chaining of observation and distribution and

[5] Incidentally, note that in the agreement assessment, implying a decision, uncertainty plays a fundamental role.

[6] Note that we have said that a theory could be seen just as a very general model, so we consistently speak of a representational theory.

describing the observation through a parametrical conditional probability distribution. Again, this model may be interpreted as describing the measurement process in general or as a framework leading the development of specific targeted models for the specific cases. From a foundational standpoint, it is important to note that often models in science and in engineering are formulated by identifying a set of variables and assuming a set of equations containing these variables.

When we model a measurement process by an input-output model, where the input is the value of the measurand, x, the output is the measurement value, $\hat{x}$, and the relation linking them is provided by the formula (7), we are following the pattern just seen. This is why we have called this description a model of the measurement process in Ref [16]. But if this is the “model”, consider now the description $a \mapsto \hat{x}$, provided by the formula (14). With respect to the previous one, this is somehow a meta-modelling, since it involves a relation (mapping) between characteristics of objects and variables describing them, rather them from variables to other variables. It could be objected that the object characteristics, when they appear in our discourse, become abstract entities as well, as terms of a discussion. This is true, but this description is still structurally different from the previous one (this is what we meant with the term meta–modelling) and this structural difference holds for measurement in general.

This is a very remarkable feature of measurement modelling emphasising the fundamental role of measurement in science, pointed out also, by different arguments, by other authors [5, 20].

7. CONCLUSIONS

In this paper we have discussed some key concepts and terms in measurement, which are particularly significant in the perspective of developing a general theory of measurement virtually applicable in all the domains of science. We have considered the importance of the measure value notion, as a necessary term for clarifying the meaning of measurement comparing its interpretation in a deterministic and in a probabilistic theory. We have discussed the notions of measuring system and measurement process providing a general probabilistic model for them. We dealt with the difference between the measure and the measurement value showing how this allows establishing a taxonomy of uncertainty sources. We examined the notion of measurement model in its association with the measurand and the measurement process. We outlined the special character of measurement modelling, showing how it reveals, from a special perspective, the foundational role of measurement for science and engineering.

We conclude by outlining once again the need of promoting theoretical and foundational studies in measurement, since this discipline, though (or, perhaps, because of) dealing mainly with experimentation, poses theoretical questions which are not less challenging than those arousing in other, apparently more theoretically oriented, disciplines.

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