Rational Preferences and Rationalizable Choices, Necessary and Possible Rankings in Decision Making under Uncertainty

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WHY A TALK ON DECISION MAKING UNDER UNCERTAINTY IN A MCDA MEETING?

- Because decision making under uncertainty can be seen as specific type of multiple criteria decisions in which the criteria are the states of nature
- Because preference structures considered in the recent research on decision making under uncertainty are analogous to preference structures recently used in MCDA
- Because recent considerations on axiomatic basis for decision making under uncertainty can be interesting for MCDA
- Because probably there is some space to develop models putting together decision making under uncertainty and MCDA

PLAN OF THE TALK

- one-preference $\succsim$ and two-preference $(\succsim^*, \succsim^\circ)$ models of decision making under uncertainty
- relations with multi-criteria decision aiding
- rational preferences and rationalizable choices: an axiomatization

EXPECTED UTILITY

- one preference relation $\succsim$
- $\succsim$ complete and transitive
- one single probability p

$$f \succsim g \Longleftrightarrow \int u(f)\, dp \geq \int u(g)\, dp$$

von Neumann and Morgenstern (1944), Savage (1954), and Anscombe and Aumann (1963)

COMPLETENESS?

> *We have conceded that one may doubt whether a person can always decide which of two alternatives ... he prefers. If the general comparability assumption is not made, a mathematical theory ... is still possible ...*
> *von Neumann and Morgenstern (1944)*

INTROSPECTION: No difficult choice between *only* two alternatives would survive if we already had a unique pre-existing complete preference in our brain (subjective states)

MULTIPLE PRIORS I

- one preference relation $\succsim$
- $\succsim$ reflexive and transitive (not complete)
- a set C of probabilities p

$$f \succsim g \iff \int u(f)\, dp \geq \int u(g)\, dp \qquad \text{for all } p \in C$$

Bewley (1986, published 2002, related to Aumann 1962)

MULTIPLE PRIORS II

- one preference relation $\succsim$
- $\succsim$ complete and transitive
- a set C of probabilities p

$$f \succsim g \iff \min_{p \in C} \int u(f)\, dp \geq \min_{p \in C} \int u(g)\, dp$$

Gilboa and Schmeidler (1989)

(A Waldean solution to the Ellsberg paradox)

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- more on two-preference models and literature review at the end ...

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Gilboa, Maccheroni, Marinacci and Schmeidler (2010)

MULTIPLE PRIORS IV

- one preference relation $\succsim$
- $\succsim$ complete ($\sim$ not transitive)
- a set C of probabilities p

$$f \succsim g \Longleftrightarrow \int u(f)\,dp \geq \int u(g)\,dp \qquad \text{for some } p \in C$$

Lehrer and Teper (2011)

(p rationalizes the choice of f from $\{f, g\}$ in the obvious game against nature)

MULTIPLE PRIORS V

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THIS PAPER

Basic concepts of Multiple Criteria Decision Aiding

- $A = \{a, b, c, ...\}$ set of alternatives
- $G = \{g_1, g_2, ..., g_n\}$ set of criteria $g_i : A \rightarrow \mathbb{R}$ such that

$$a \succsim^i b \iff g_i(a) \geq g_i(b)$$

- Dominance

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If $a \succsim^G b$ the choice of a from $\{a, b\}$ is rational in a very intuitive sense

what if neither $a \succsim^G b$ nor $b \succsim^G a$?

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$$\succsim^{*} = \bigcap_{m \in M(GO)} \succsim_m \quad \text{and} \quad \succsim^{\circ} = \bigcup_{m \in M(GO)} \succsim_m$$

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Jacquet-Lagrèze and Siskos (1982), Greco, Mousseau, Słowiński (2008), Giarlotta and Greco (2013)

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THIS PAPER

SETUP (ANSCOMBE-AUMANN)

- (S, Σ) a measurable space of *states of the world*
- X a convex set of *consequences*
- Δ the set of *probabilities* on Σ

  (with the event-wise convergence topology)
- F the set of all *acts*: simple measurable functions from S to X
- $\succsim^*$ and $\succsim^\circ$ two binary relations on F

BASIC AXIOMS

¿ reasoning templates/guidelines OR descriptions of behavior ?

Basic Conditions (BC)

Reflexivity: $f \succsim f$.

Monotonicity: $f(s) \succ g(s)$ *for all* $s \in S$ *implies* $f \succ g$

Continuity: $\{\lambda \in [0,1] : \lambda e + (1-\lambda) f \succsim \lambda g + (1-\lambda) h\}$ *is closed*

Non-triviality: there exist constant f *and* g *in* F *such that* $f \succ g$

AXIOMS FOR RATIONALITY

C-Completeness, Transitivity, and Independence

C-Completeness: if f and g are constant, then either $f \succsim^* g$ or $g \succsim^* f$

Transitivity: $f \succsim^* g$ and $g \succsim^* h$ imply $f \succsim^* h$

Independence: $f \succsim^* g$ implies $\lambda f + (1-\lambda)h \succsim^* \lambda g + (1-\lambda)h$ for all λ in $(0,1)$

AXIOMS FOR RATIONALIZABILITY

Completeness, C-Transitivity, and C-Independence

Completeness: either $f \succsim^{\circ} g$ or $g \succsim^{\circ} f$

C-Transitivity: if f, g, and h are constant, $f \succsim^{\circ} g$ and $g \succsim^{\circ} h$ imply $f \succsim^{\circ} h$

C-Independence: if h is constant, $f \succsim^{\circ} g$ implies $\lambda f + (1-\lambda)h \succsim^{\circ} \lambda g + (1-\lambda)h$ for all λ in $(0,1)$

INTERTWINING ...

Transitive Consistency: *If either* $f \succsim^* g \succsim^\circ h$ *or* $f \succsim^\circ g \succsim^* h$, *then* $f \succsim^\circ h$

Possibility: *If* $g \not\succsim^* f$, *then* $f \succsim^\circ g$. *(*$g \succsim^* f$ *or* $f \succsim^\circ g$*)*

REPRESENTATION THEOREM

The following are equivalent for $(\succsim^*, \succsim^\circ)$.

- $\succsim^*$ satisfies the BC, C-Completeness, Transitivity, and Independence, $\succsim^\circ$ satisfies BC, Completeness, C-Transitivity, and C-Independence, jointly $(\succsim^*, \succsim^\circ)$ satisfy Transitive Consistency and Possibility;
- there exist a non-empty closed and convex set C of probabilities on Σ and a non-constant affine $u : X \to \mathbb{R}$ such that, for any $f, g \in F$,

$$f \succsim^* g \iff \int u(f)\, dp \geq \int u(g)\, dp \quad \text{for all } p \in C$$

and

$$f \succsim^\circ g \iff \int u(f)\, dp \geq \int u(g)\, dp \quad \text{for some } p \in C.$$

In this case, C is unique and u is unique up to positive affine transformations.

TWO PREFERENCE MODELS

- in/completeness of beliefs/tastes
  - Nehring (2008)
- psychological and revealed preferences
  - Mandler (2005)
  - Danan (2006)
- status quo bias completion
  - Masatlioglu and Ok (2005)
- choice deferral
  - Danan and Ziegelmeyer (2006)
  - Kopylov (2009)
    - + $f \succsim^{\circ} g$ if the agent is willing to choose f over g when no other alternatives are feasible
    - + $f \succsim^{*} g$ if the agent is willing to choose f over g even if she has the option to postpone this choice

FUTURE RESEARCH

- the good news is that both $\succsim^*$ and $\succsim^\circ$ can be elicited from behavior (Nishimura, 2014, Cerreia-Vioglio and Ok, 2015)
- in particular in Cappelli, Corrente, Greco, Maccheroni, Marinacci (2015) we are investigating the possibility of computing both $\succsim^*$ and $\succsim^\circ$ in multicriteria decision making under uncertainty (a relatively new and promising field of MCDA)