Interactive Evolutionary Multiobjective Optimization with Choquet Integral Preference Model Derived by Robust Ordinal Regression

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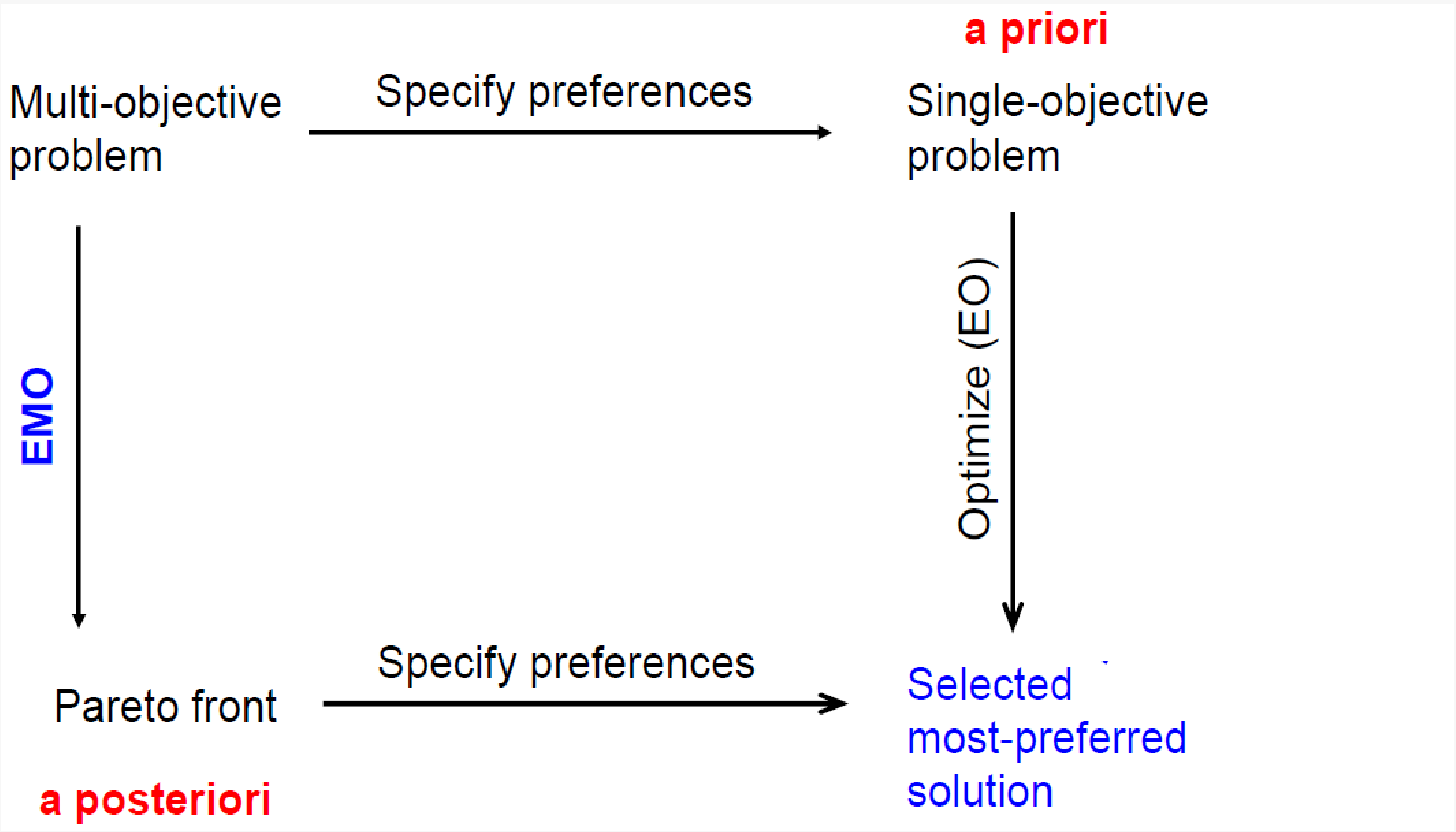
81st Meeting of the EURO WG on MCDA, Annecy, 26-28 March 2015

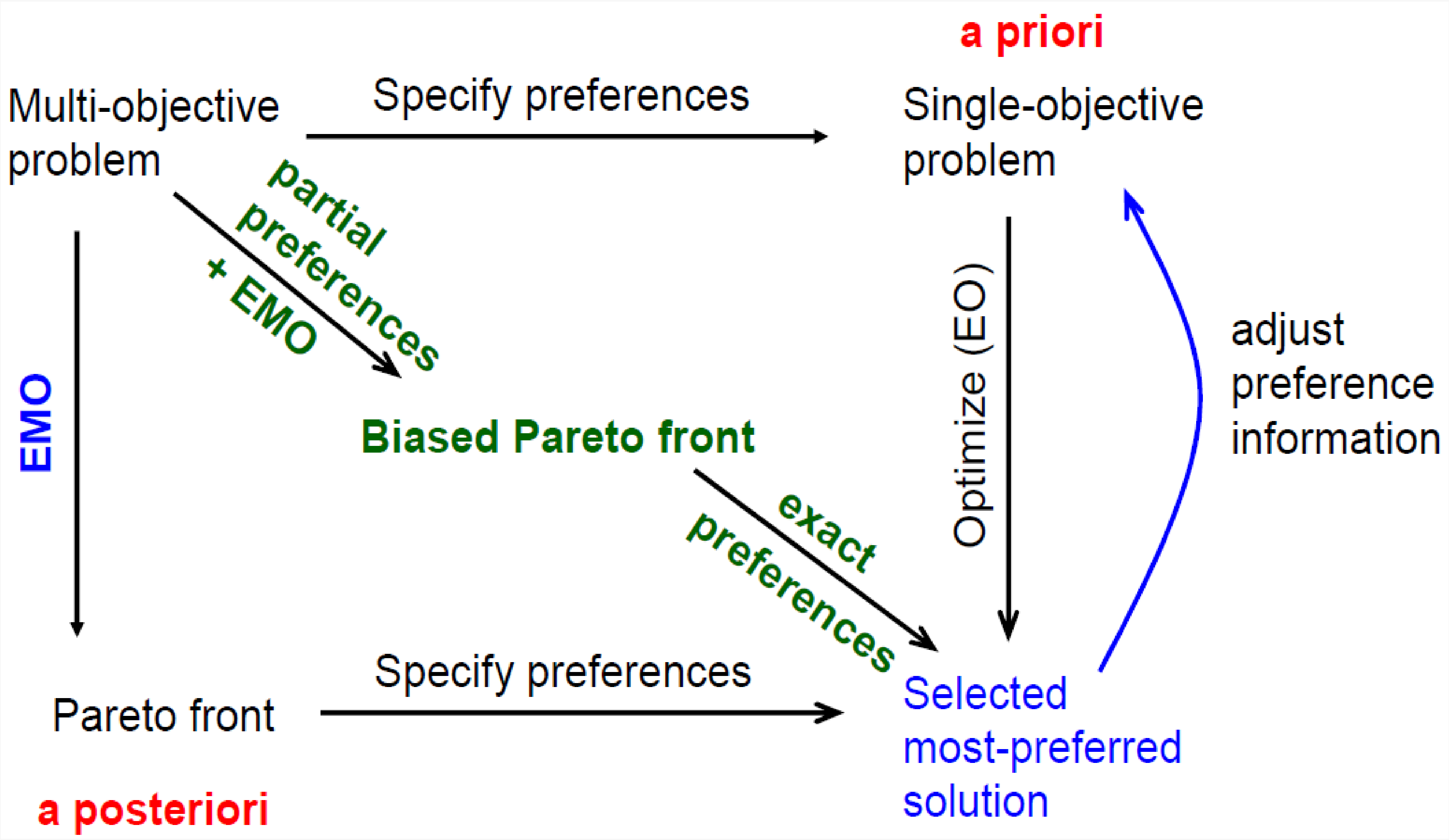
Outline

- Interactive Multiobjective Optimization
- Learning user's preferences from user-machine interactions
- Preference learning in EMO
- The NEMO framework
- The NEMO-II-Ch method
- Some empirical results
- Conclusions

Multiobjective Optimization & EMO

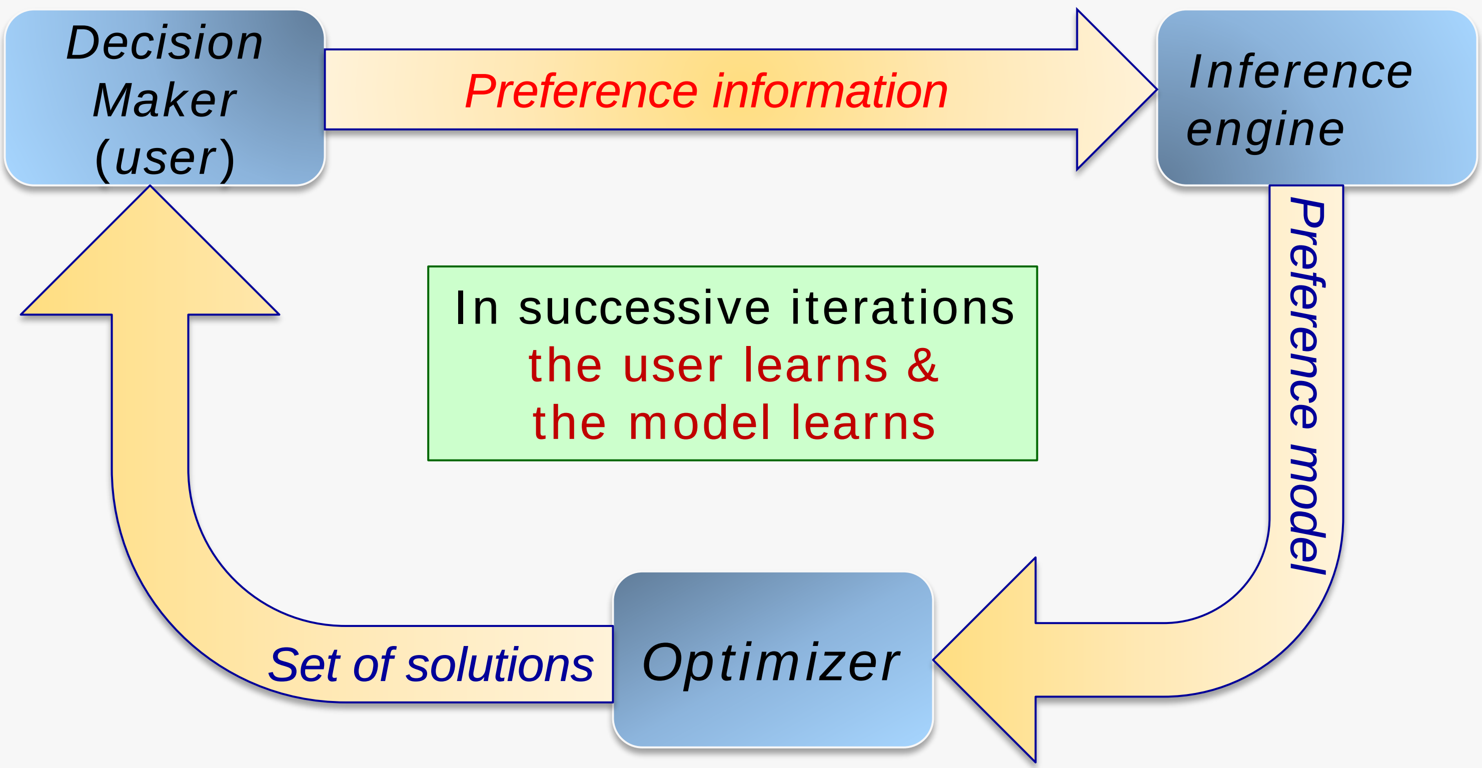


Interactive Multiobjective Optimization & EMO



Interactive optimization

- DM looks at intermediate results from optimization
- DM provides preference information
- Optimizer uses DM's preferences to focus the search on most promising solutions

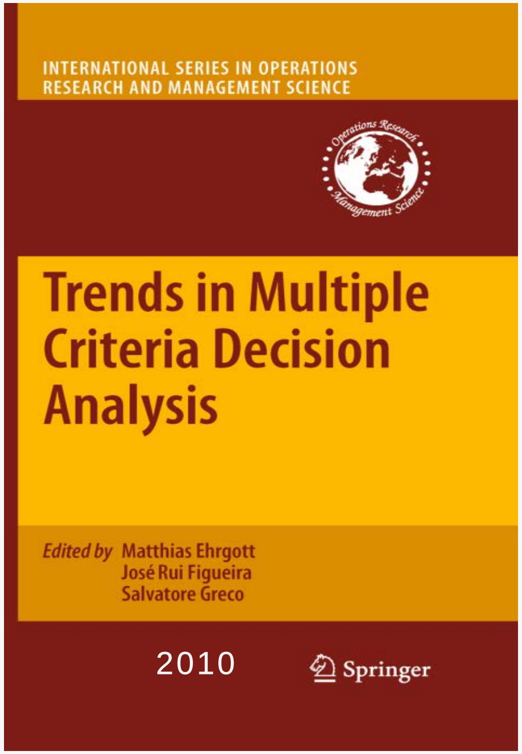


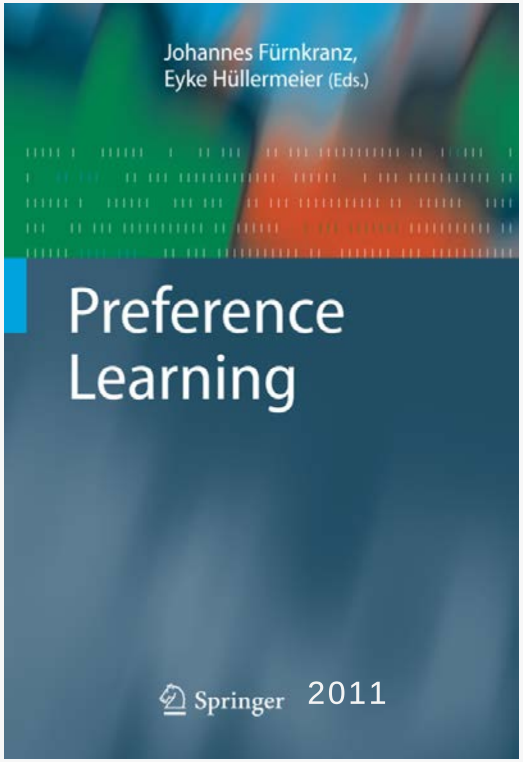
What and from what information the model (machine) can learn ?

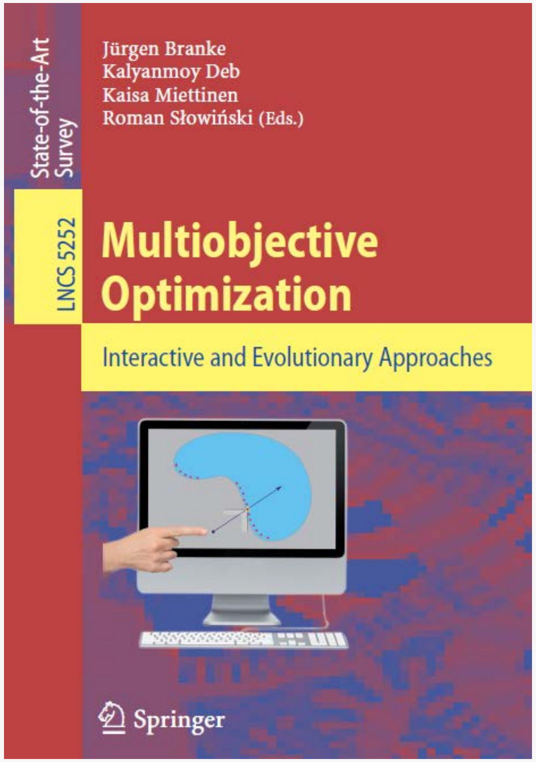
- Model learning is a concept which underlines an evolution of the model in view of facts observed through sensors in an external world
- A model is implemented as a computer program on a machine, hence the term machine learning is often used instead of model learning
- Machine, or model, learning is the ability of a computer program to improve its performance by learning from data
- The model relates an output (preference structure) with an input (preference information), either analytically, using a *function*, or logically, using *decision rules* or *trees*

Learning user's preferences from user-machine interactions

- *Preference learning* is about inducing predictive preference models from empirical data (falls into a broad term of *regression*)
- Constructive preference learning in MCDA, versus stochastic preference learning in Machine Learning

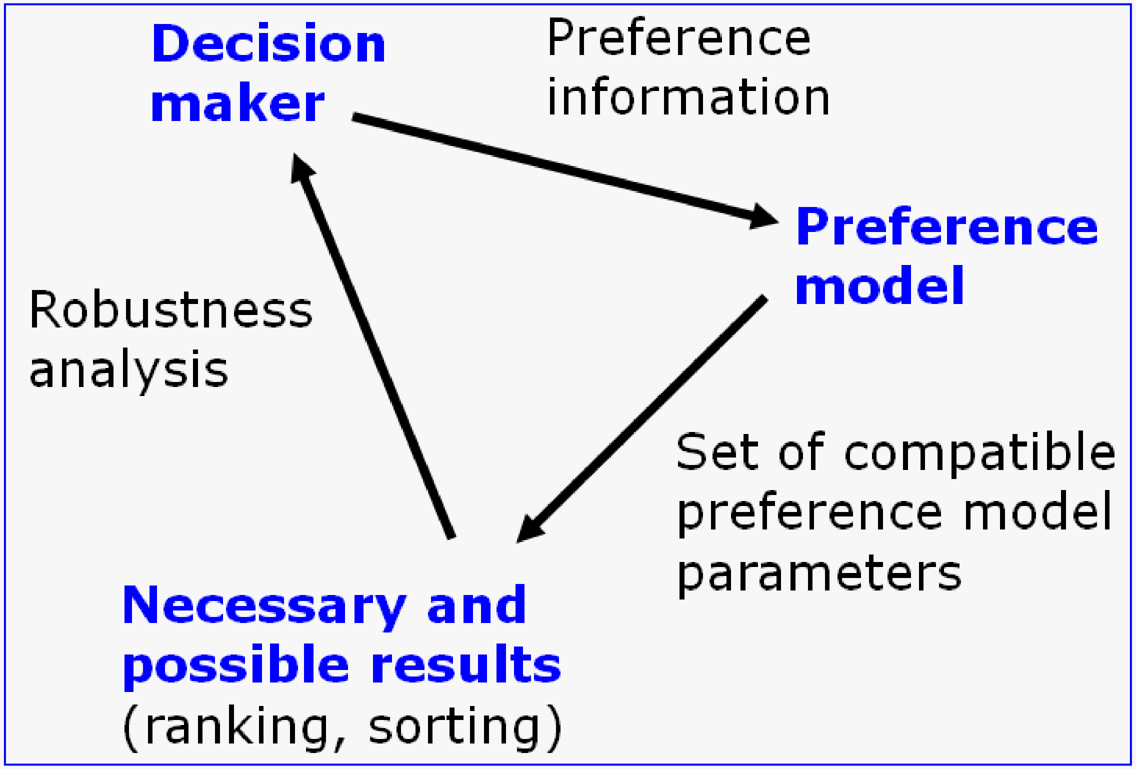






Interactive optimization with Robust Ordinal Regression

- Ordinal regression paradigm emphasizes the discovery of intentions as an interpretation of actions rather than as a priori position (disaggregation approach)
- Robust Ordinal Regression in a loop: preference elicitation with constructive learning
- Results are robust, because they take into account partial preference information



Interactive optimization with Robust Ordinal Regression

Input

- Pairwise comparisons of solutions
- Best (or worst) solution out of a set
- Ranking of several solutions
- Ordinal or cardinal intensity of preference for pairs of solutions
- Sorting of solutions into quality classes
- ...

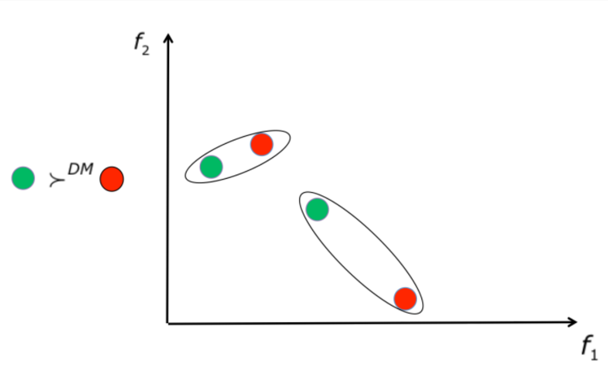
Output

- Value function
- Outranking relation
- Artificial neural network
- Decision rules
- Decision trees
- ...

How complex should the model be?

- Model too simple

  ➡ not able to represent user's preferences



- Example: linear model unable to capture preference information

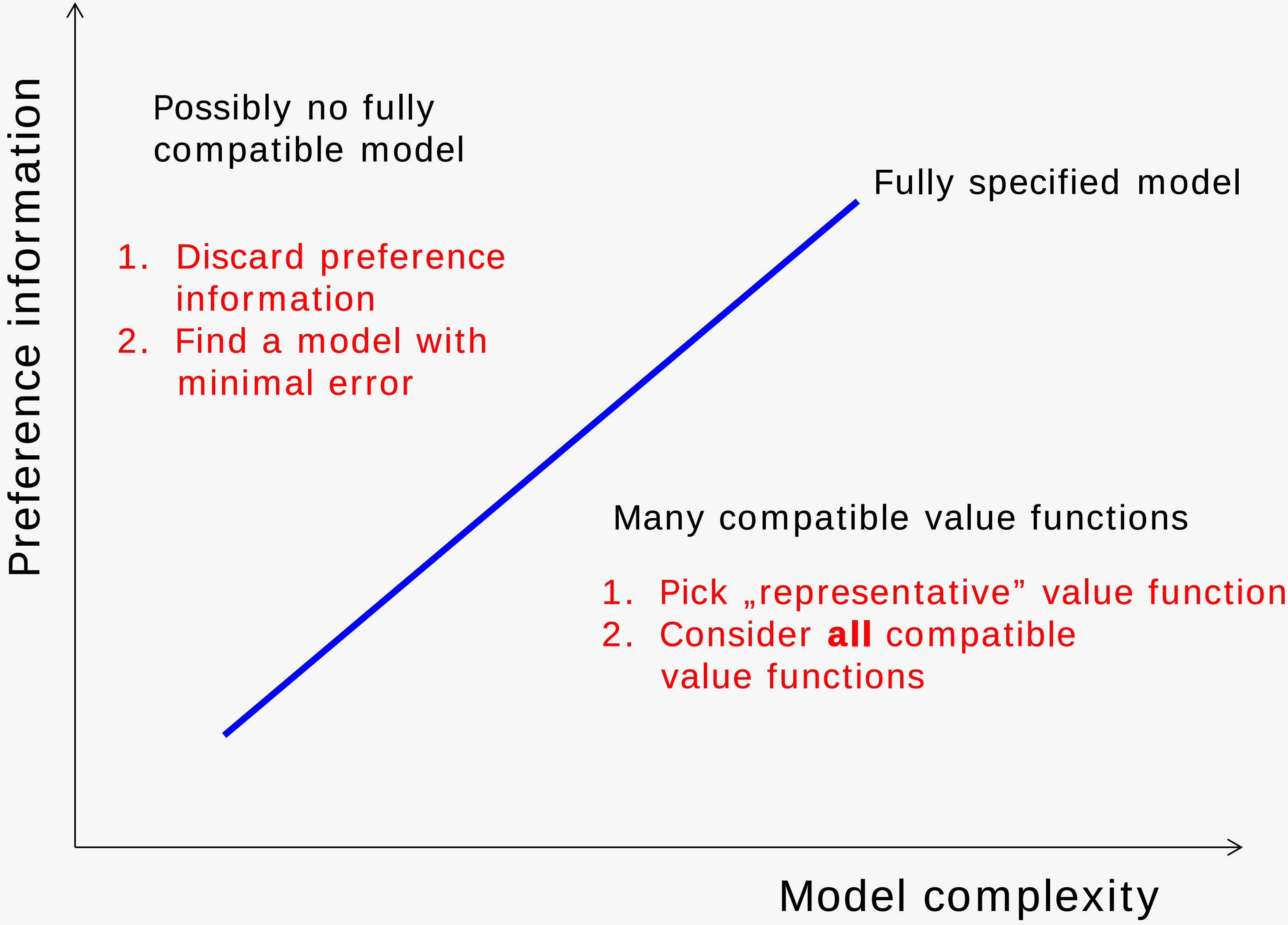
- Model too complex/flexible

  ➡ no generalization power, all solutions enter only one front, takes very long to learn all the parameters

- Example: Dominance relation, general additive model with monotonic marginal value functions

„Everything should be made as simple as possible – but not simpler" [Albert Einstein]

Preference information and model complexity



Many compatible value functions

1. Pick a **„representative"** value function
   - Most discriminative
     (for reference solutions or for the necessary relation)
   - Minimize bends
   - Maximize total utility
2. Consider **all** compatible value functions
   - EMO can naturally deal with incomparability
   - Necessary and possible preference relations

The NEMO framework [Branke, Greco, Słowiński, Zielniewicz 2009, 2010, 2014] [Branke, Corrente, Greco, Słowiński, Zielniewicz 2014]

- NEMO integrates ROR into NSGA-II
- ROR implemented in
  - UTA^{GMS} [Greco, Mousseau, Słowiński 2009]
  - GRIP [Figueira, Greco, Słowiński 2009]
- Preference model:
  - Additive value function $U(a) = \sum_{i=1}^{n} u_i[f_i(a)]$
  - Monotonic marginal value functions u_i
- Necessary preference relation or representative value function is used to rank solutions in the current population
- No scaling of objectives is necessary – NEMO handles heterogeneous objectives

The NEMO framework [Branke, Greco, Słowiński, Zielniewicz 2014]

- Integrates Robust Ordinal Regression into EMO
- NEMO-0
  - Learn a „representative” value function
  - Use „representative” value function to rank individuals with the same Pareto rank

NEMO-0: a single compatible value function is used to rank solutions in the population

- Every *q* iterations the DM elicits preferences by comparing pairwise some non-dominated solutions in the current population
- Determine the dominance ranking (partial order of solutions obtained by iterative removing of a non-dominated front)
- Within each non-dominated front, rank individuals according to a representative value function
- Different representative value functions:
  - MDVF: Most Discriminative Value Function
  - MSCVF: Min Slope Change Value Function
  - MSVF: Max Sum of Value Function Scores (total utility)

The NEMO framework [Branke, Greco, Słowiński, Zielniewicz 2014]

- Integrates Robust Ordinal Regression into EMO
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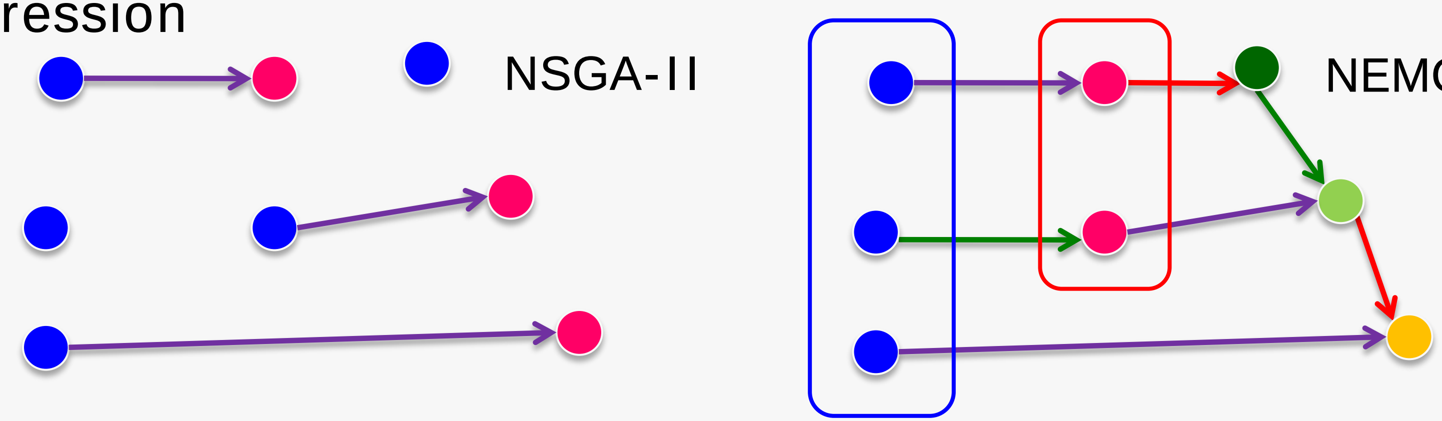
  - Learn a „representative" value function
  - Use „representative" value function to rank individuals with the same Pareto rank
- NEMO-I [Branke, Greco, Słowiński, Zielniewicz 2009, 2010]
  - Replaces dominance relation by pairwise necessary preference relation
  - $O(n^2)$ LPs to solve

NEMO-I: the whole set of compatible value functions is considered

- The dominance relation used in NSGA-II to rank solutions is replaced by the necessary preference relation of robust ordinal regression



- A representative value function is used in the crowding distance:

$$CD(x) = \sum_{i=1}^{n} \left| u_i\left(y_i^x\right) - u_i\left(z_i^x\right) \right| - \left| U\left(y^x\right) - U\left(z^x\right) \right|$$

where U is a representative value function, u_i are its marginal value functions, y_i^x, z_i^x are left and right neighbors of x wrt u_i, and y^x, z^x are vectors composed of y_i^x, z_i^x, respectively

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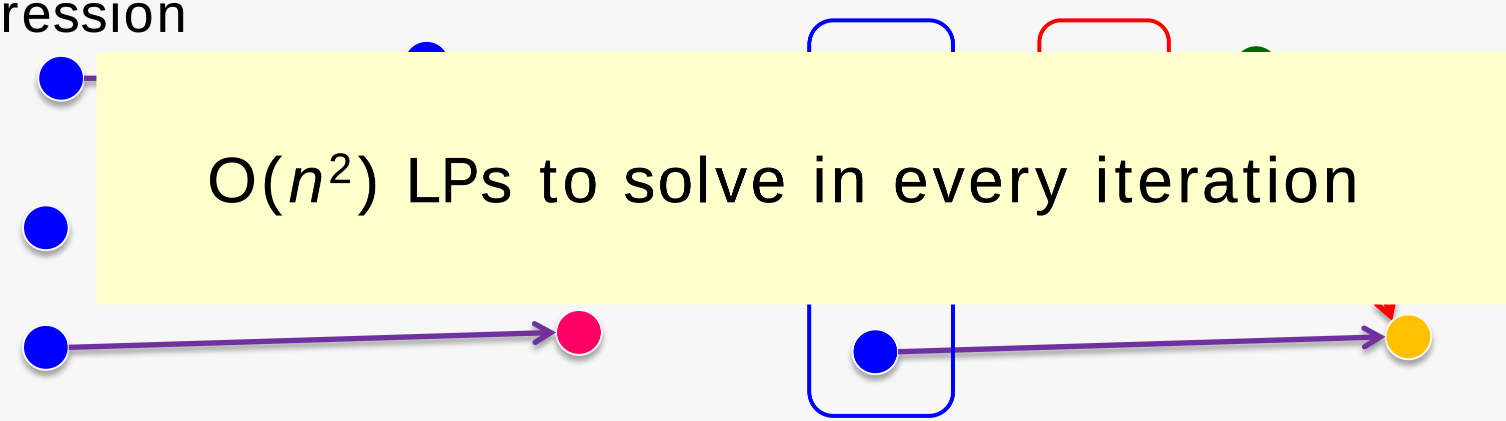
- The dominance relation used in NSGA-II to rank solutions is replaced by the necessary preference relation of robust ordinal regression



O(n^2) LPs to solve in every iteration

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The NEMO framework [Branke, Greco, Słowiński, Zielniewicz 2014]

- Integrates Robust Ordinal Regression into EMO
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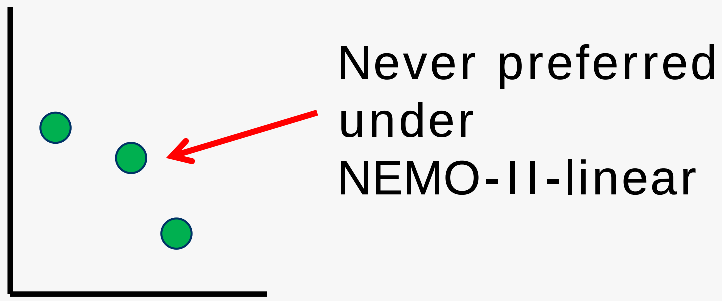
- NEMO-I [Branke, Greco, Słowiński, Zielniewicz 2009, 2010]
  - Replaces dominance relation by pairwise necessary preference relation
  - $O(n^2)$ LPs to solve
- NEMO-II
  - A solution is „possibly preferred” if there is a compatible value function that would prefer this solution over all others
  - Only O(n) LPs to solve



Approaches to learning a single preference function

- Phelps&Köksalan [2003]

  $f(x)=ax_1^2+bx_2^2$, most discriminative

- Deb, Sinha, Korhonen, Wallenius [2010]

  Polynomial value function model, most discriminative

- Todd and Sen [1999]

  Artificial Neural Network

- Battiti and Passerini [2010]

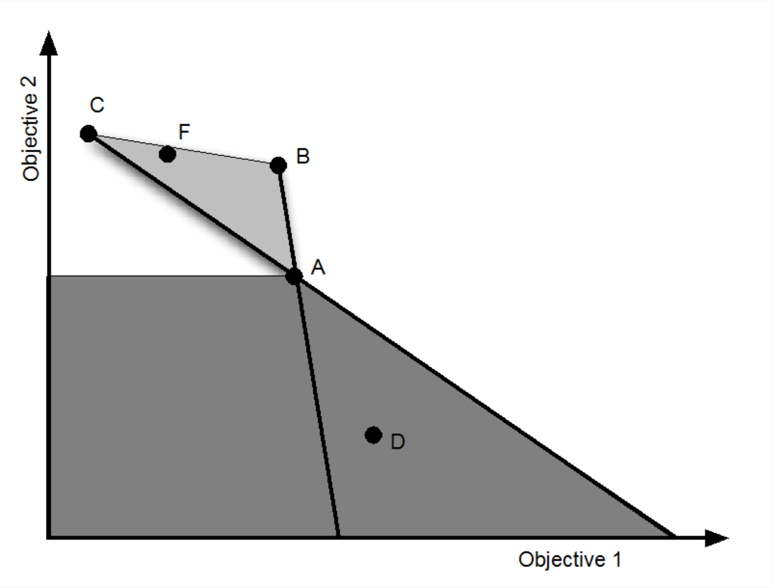
  Support Vector Machine (with cross-validation to determine most appropriate kernel)

- Branke, Greco, Słowiński and Zielniewicz [2014]

  Additive monotonic value function, maximum total utility

Approaches to learning a set of preference functions

- Jaszkiewicz [2007]
  Set of compatible linear preference functions, samples one in each generation
- Greenwood, Hu and D'Ambrosio [1997]
  Linear preference model, necessarily dominated solutions are considered inferior (equivalent to NEMO-I with linear preference model)
- Fowler et al [2010]
  Quasi concave value functions
- Branke, Greco, Słowiński and Zielniewicz [2009,2010]
  NEMO-I with piecewise-linear and additive monotonic preference model



Recent work: NEMO-II-Choquet

- Use Choquet integral as preference model
  - Well-accepted model in decision theory
  - Allows to model interaction between objectives
- Adapt complexity of preference model to complexity of preferences
  - Start with linear model
  - Switch to 2-additive Choquet once no linear compatible value function can be found

Choquet integral (1)

The Choquet integral [Choquet 1954] substitutes the usual weighted sum by a weight for each subset of the criteria, e.g.:

- $\mu(\emptyset)=0$,
- $\mu(\{\text{Mathematics}\}) = \mu(\{\text{Physics}\}) = 0.45$,
- $\mu(\{\text{Literature}\})=0.3$,
- $\mu(\{\text{Mathematics, Physics}\})=0.5$,
- $\mu(\{\text{Mathematics, Literature}\})=\mu(\{\text{Physics, Literature}\})=0.9$,
- $\mu(\{\text{Mathematics, Physics, Literature}\})=1$.

This permits to take into account the synergy between criteria

Choquet integral (2)

- It is based on capacity μ defined over set $N=\{1,2,\ldots,n\}$ of criteria:

$$\mu : 2^N \rightarrow [0,1]$$

  - Monotonicity condition:

$$\mu(S) \leq \mu(T), \;\; \forall S,T : \; S \subseteq T (\subseteq N)$$

  - Boundary condition:

$$\mu(\varnothing) = 0, \quad \mu(N) = 1$$

Choquet integral (3)

Given n evaluations (gain-type) $f_1,\ldots,f_n$ with $f_i \geq 0, \quad \forall i = 1,\ldots,n,$

the Choquet integral of $(f_1,\ldots,f_n)$ is computed as follows:

$$Ch_\mu(f_1,\ldots,f_n) = \sum_{i=1}^{n}\left(f_{(i)} - f_{(i-1)}\right)\mu\left(F_{(i)}\right)$$

where:

$f_{(0)} = 0,$

$(\,\cdot\,)$ index permutation : $f_{(i-1)} \leq f_{(i)},\ \ i = 1,\ldots,n$

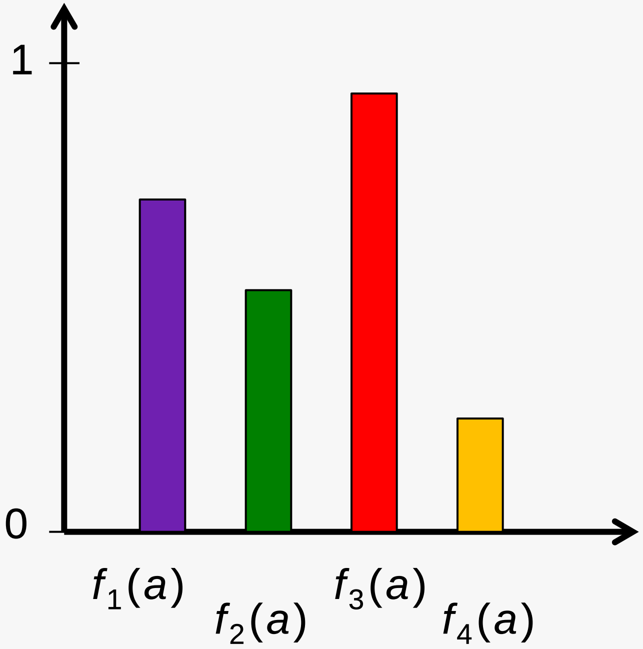
$F_{(i)} = \left\{f_{(i)},\ldots,f_{(n)}\right\}$

Additive vs. non-additive aggregation

- Instead of weights w_i for each objective $f_i \in F$ in a weighted sum:
  $\mu(F')$ – joint weight of criteria from a subset $F' \subseteq F$

- $\mu : 2^F \to [0, 1]$ – non-additive measure (capacity):
  - $\mu(\emptyset) = 0,\ \mu(F) = 1$
  - for $F'' \subset F' \subseteq F$, $\mu(F'') \leq \mu(F')$
  - in general, $\mu(F'' \cup F') \neq \mu(F'') + \mu(F')$
  - positive interaction (synergy): $\mu(F'' \cup F') > \mu(F'') + \mu(F')$
  - negative interaction (redundancy): $\mu(F'' \cup F') < \mu(F'') + \mu(F')$

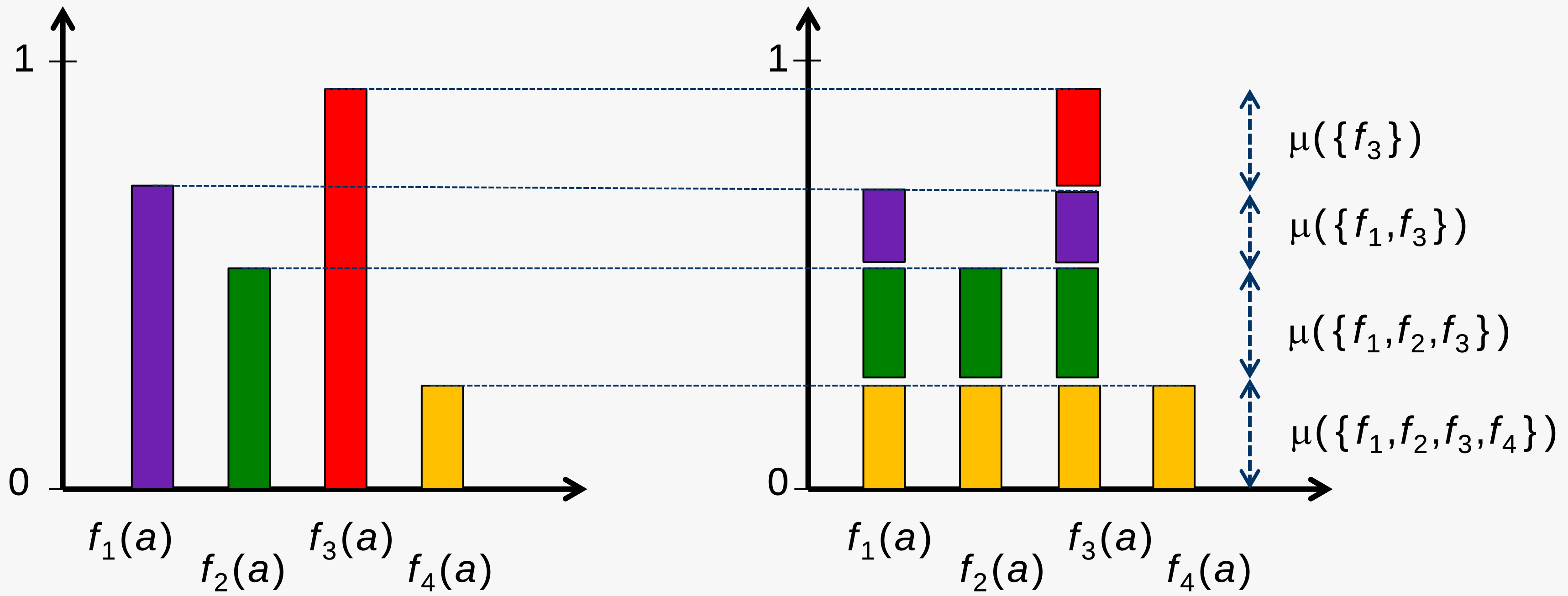
Weighted sum vs. discrete Choquet integral



Weighted sum:

$$U(a) = \sum_{i=1}^{n} k_i \; f_i(a) = \sum_{i=1}^{n} \mu(\{f_i\}) \, f_i(a)$$

Weighted sum vs. discrete Choquet integral



Weighted sum:

$$U(a) = \sum_{i=1}^{n} k_i \; f_i(a) = \sum_{i=1}^{n} \mu(\{f_i\}) \, f_i(a)$$

Choquet integral:

$$U(a) = \sum_{i=1}^{n} \mu(F_i) \left(f_{(i)}(a) - f_{(i-1)}(a) \right)$$

where $_{(\cdot)}$ is a permutation of $\{1,\ldots,n\}$, such that $0 \le f_{(1)}(a) \le f_{(2)}(a) \le \ldots \le f_{(n)}(a)$,

$F_i = \{f_{(i)},\ldots,f_{(n)}\}$, $f_{(0)} = 0$; $f_4(a) \le f_2(a) \le f_1(a) \le f_3(a) \rightarrow (1)=4,\ (2)=2,\ (3)=1,\ (4)=3$

Choquet integral (4)

- By considering the Möbius representation of 2-additive capacity μ:

$$\mu(T) = \sum_{i \in T} m(\{i\}) + \sum_{\{i,j\} \subseteq T} m(\{i,j\}), \quad \forall T \subseteq N,$$

  - monotonicity:

$$\begin{cases} m(\{i\}) \geq 0, \quad \forall i \in N, \\ m(\{i\}) + \sum_{j \in T} m(\{i,j\}) \geq 0, \quad \forall i \in N, \text{ and } \forall T \subseteq N \setminus \{i\}, \; T \neq \emptyset \end{cases}$$

  - normalization:

$$m(\emptyset) = 0, \qquad \sum_{i \in N} m(\{i\}) + \sum_{\{i,j\} \subseteq N} m(\{i,j\}) = 1$$

  - we get:

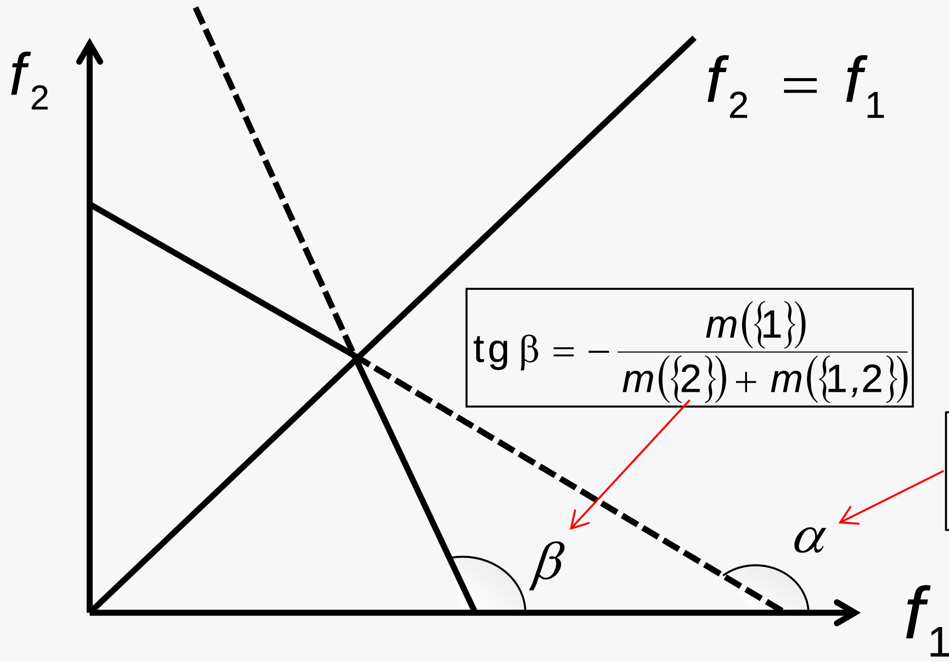
$$Ch_\mu(f_1, \ldots, f_n) = \sum_{i \in N} m(\{i\}) f_i + \sum_{\{i,j\} \subseteq N} m(\{i,j\}) \min\{f_i, f_j\}$$

A particular case of the Choquet integral: $n=2$

If $n=2$, then...

$$Ch_\mu(f_1, f_2) = m(\{1\})\, f_1 + m(\{2\})\, f_2 + m(\{1,2\}) \min\{f_1, f_2\} =$$

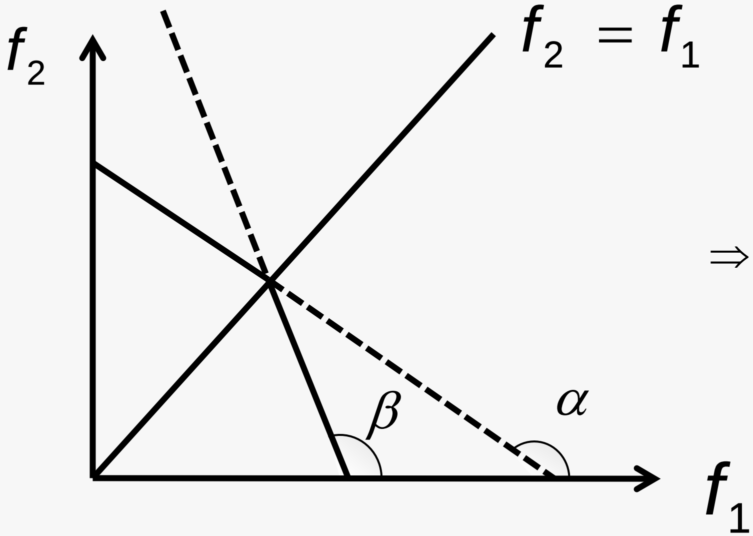
$$= \begin{cases} \left(m(\{1\}) + m(\{1,2\})\right) f_1 + m(\{2\})\, f_2 & \text{if } f_1 \le f_2 \\ m(\{1\})\, f_1 + \left(m(\{2\}) + m(\{1,2\})\right) f_2 & \text{if } f_1 \ge f_2 \end{cases}$$



$$\operatorname{tg} \beta = -\frac{m(\{1\})}{m(\{2\}) + m(\{1,2\})}$$

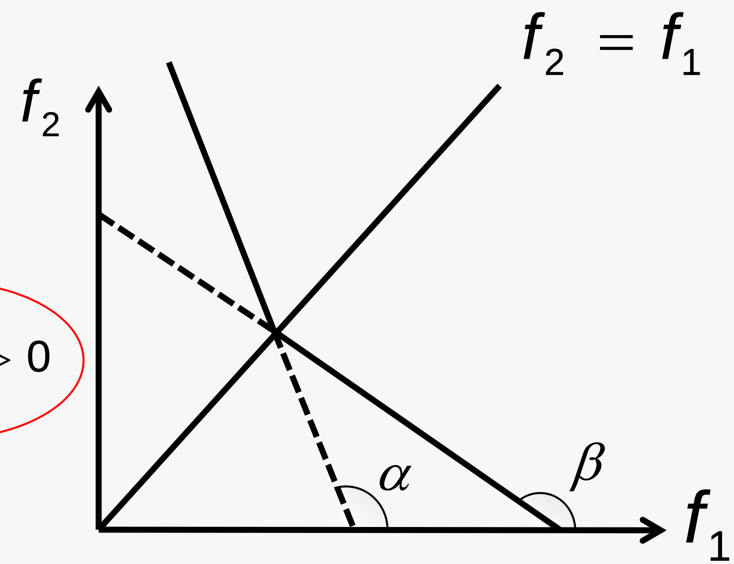
$$\operatorname{tg} \alpha = -\frac{m(\{1\}) + m(\{1,2\})}{m(\{2\})}$$

The Choquet integral isoquants ('wings')



$$\alpha > \beta \Rightarrow \mathrm{tg}\,\alpha > \mathrm{tg}\,\beta \Rightarrow$$

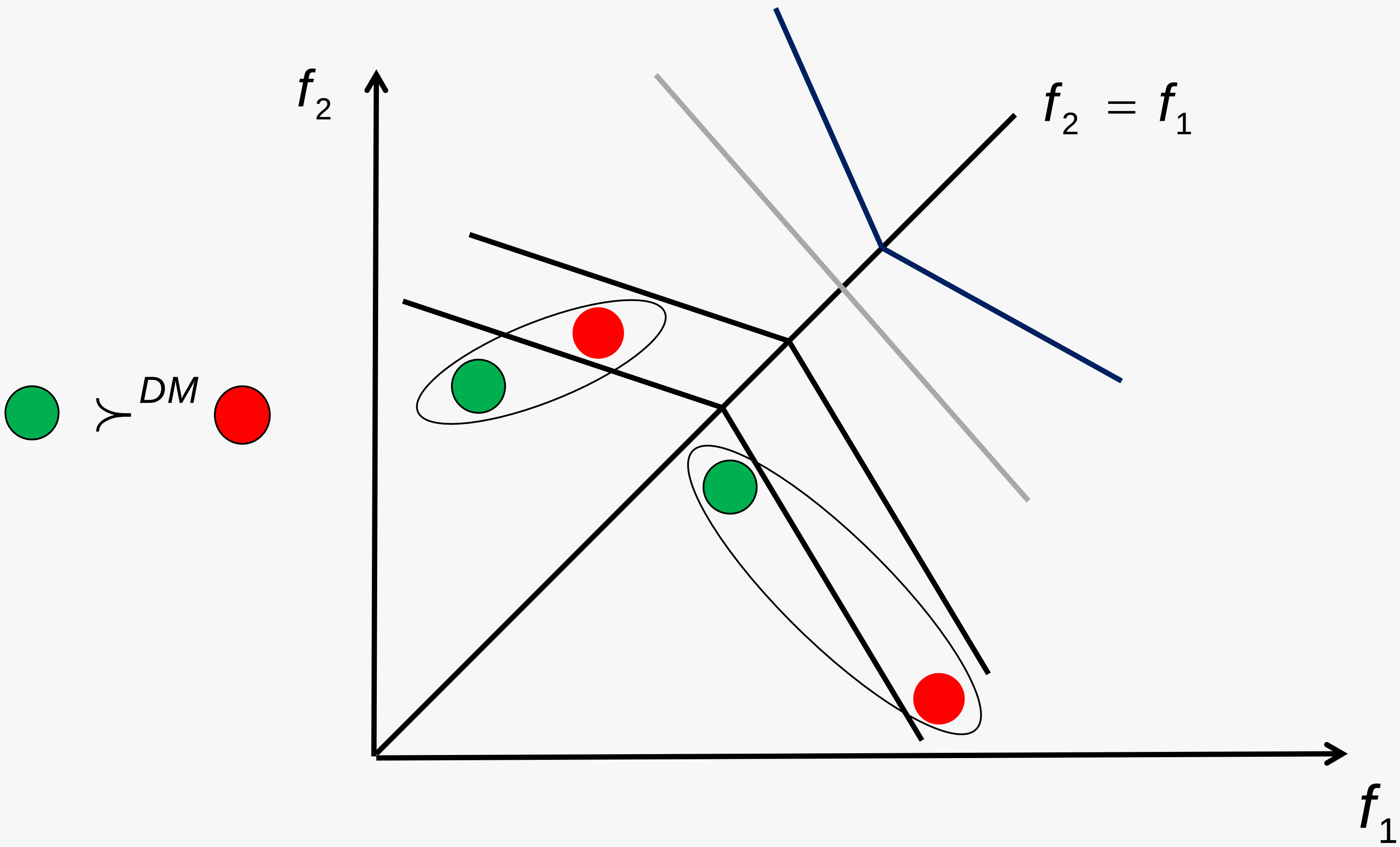
$$\Rightarrow -\frac{m(\{1\}) + m(\{1,2\})}{m(\{2\})} > -\frac{m(\{1\})}{m(\{2\}) + m(\{1,2\})} \Rightarrow m(\{1,2\}) < 0$$



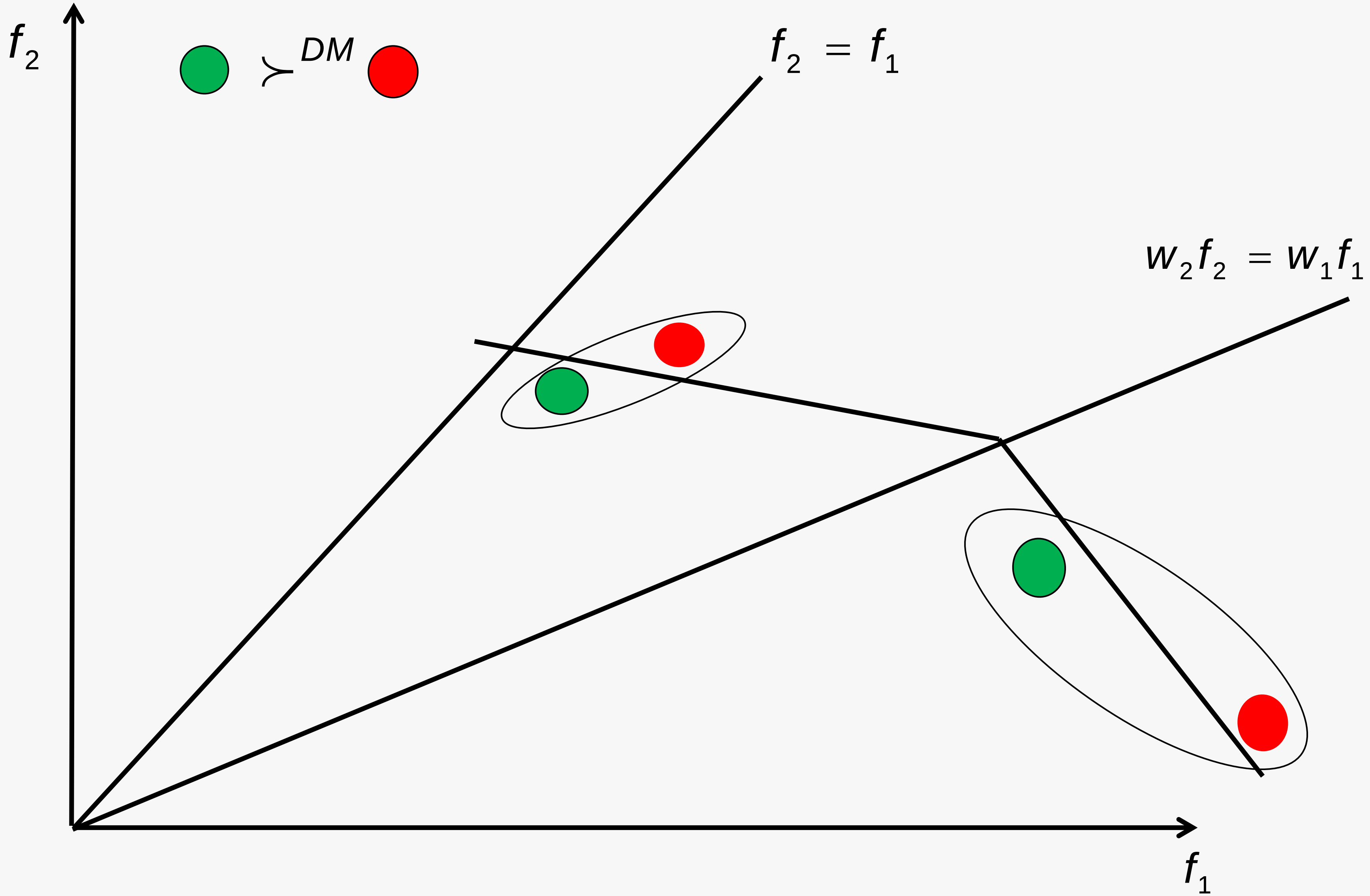
$$\alpha < \beta \Rightarrow \mathrm{tg}\,\alpha < \mathrm{tg}\,\beta \Rightarrow$$

$$-\frac{m(\{1\}) + m(\{1,2\})}{m(\{2\})} < -\frac{m(\{1\})}{m(\{2\}) + m(\{1,2\})} \Rightarrow m(\{1,2\}) > 0$$

Graphical interpretation

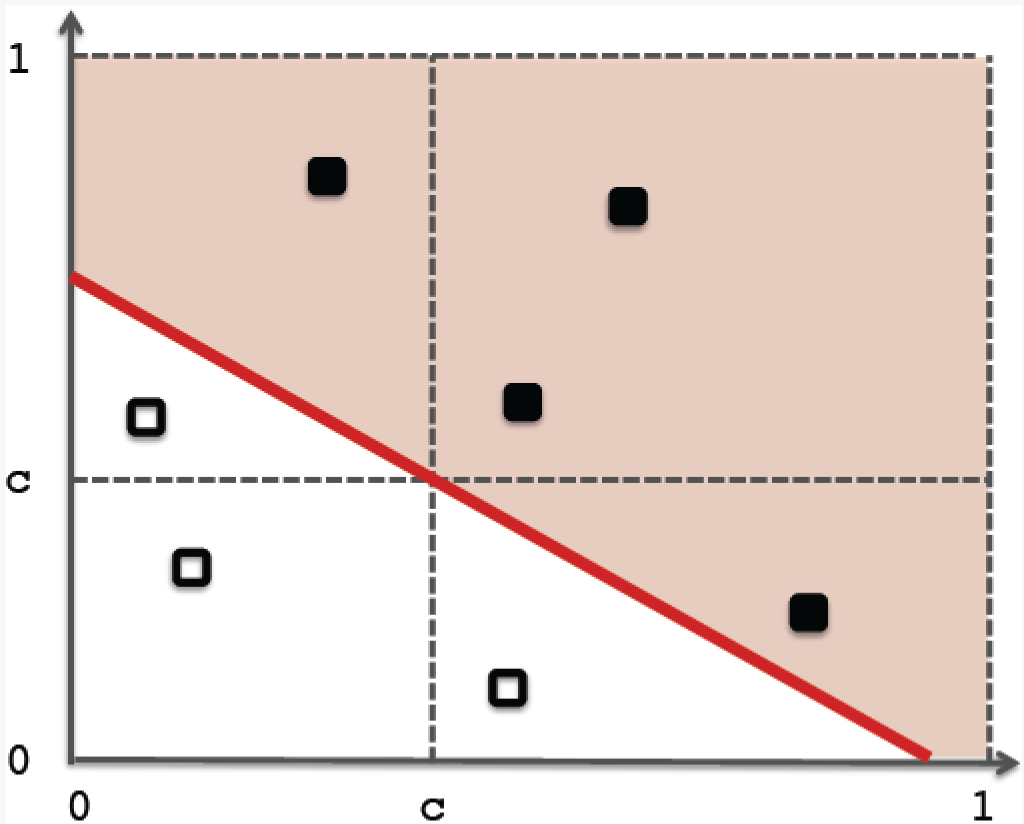


Scaling of objectives



Isoquants of the Choquet integral for two criteria – special cases

- Weighted sum (linear additive) – no interaction

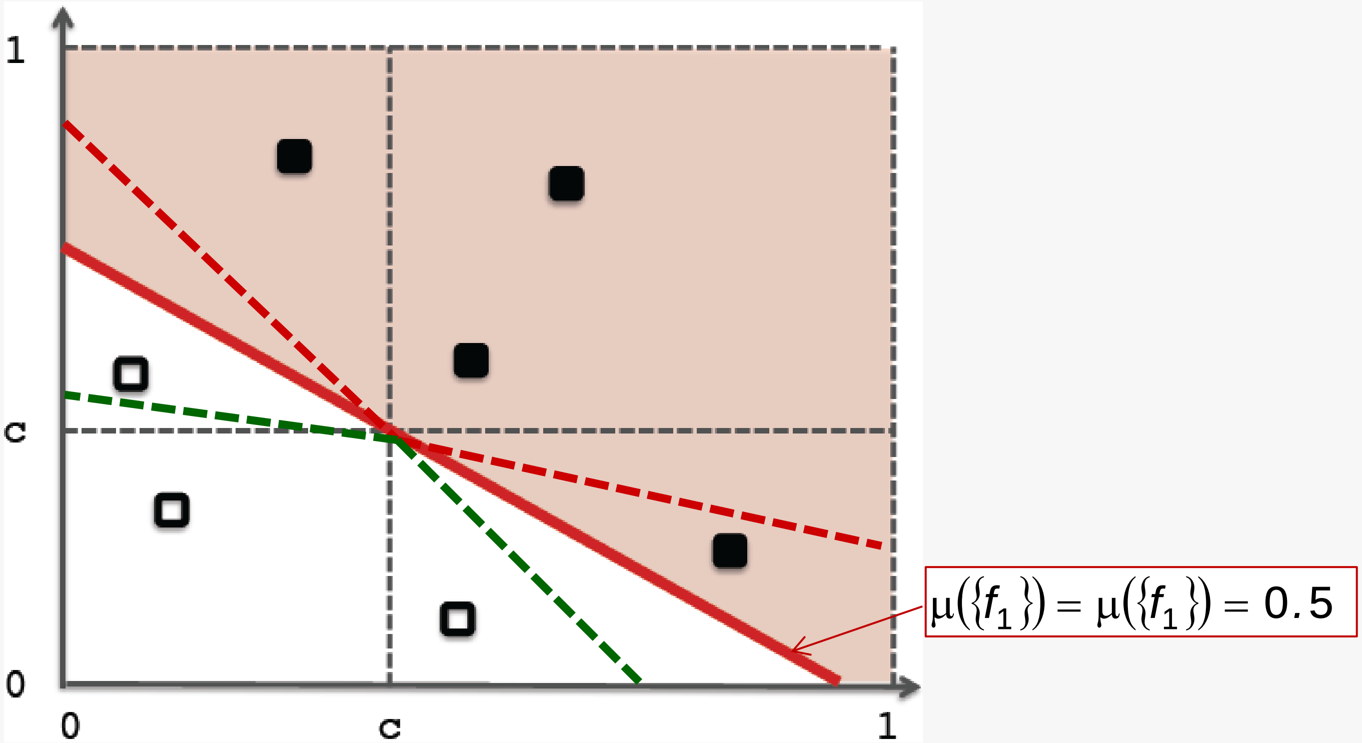


$$U(a) = \mu(\{f_1\})\, f_1(a) + \mu(\{f_2\})\, f_2(a) \geq c, \quad \mu(\{f_1\}) + \mu(\{f_2\}) = 1$$

Isoquants of the Choquet integral for two criteria – special cases

- Ordered Weighted Average (OWA) – positive interaction if $\mu(\{f_1\}) = \mu(\{f_1\}) < 0.5$

  – negative interaction if $\mu(\{f_1\}) = \mu(\{f_1\}) > 0.5$

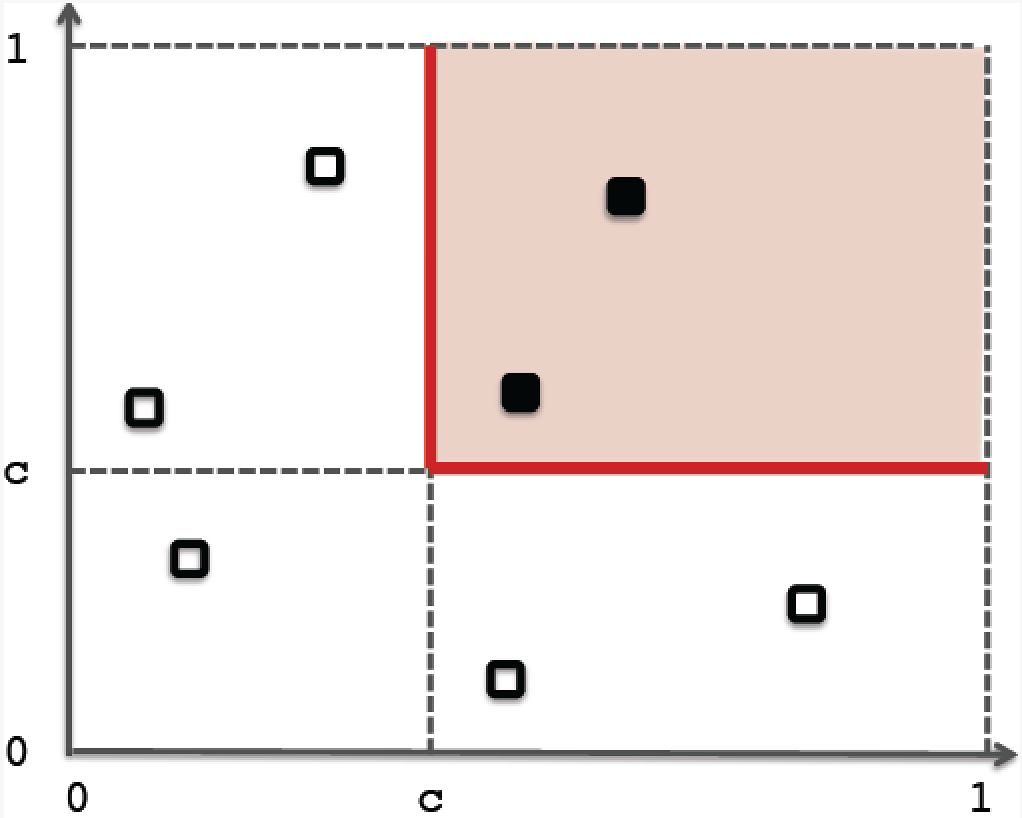


$$U(a) = k_1 f_{(1)}(a) + k_2 f_{(2)}(a) = (1 - \mu(\{f_1\}))\, f_{(1)}(a) + \mu(\{f_2\})\, f_{(2)}(a) \geq c,$$

with $\mu(\{f_1\}) = \mu(\{f_2\})$ and $\mu(\{f_1, f_2\}) = 1$

Isoquants of the Choquet integral for two criteria – special cases

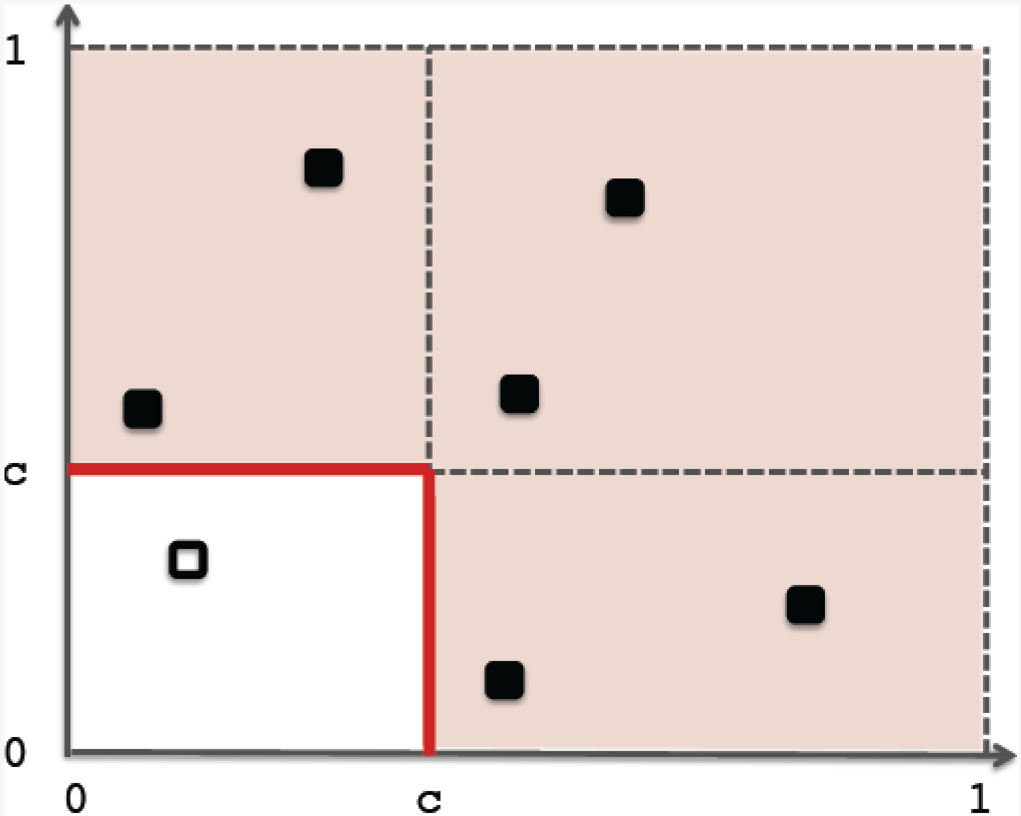
- Min – maximum negative interaction (redundancy)



$$U(a) = \min\{f_1(a), f_2(a)\} \geq c, \quad \mu(\{f_1\}) = 0, \;\; \mu(\{f_2\}) = 0, \;\; \mu(\{f_1, f_2\}) = 1$$

Isoquants of the Choquet integral for two criteria – special cases

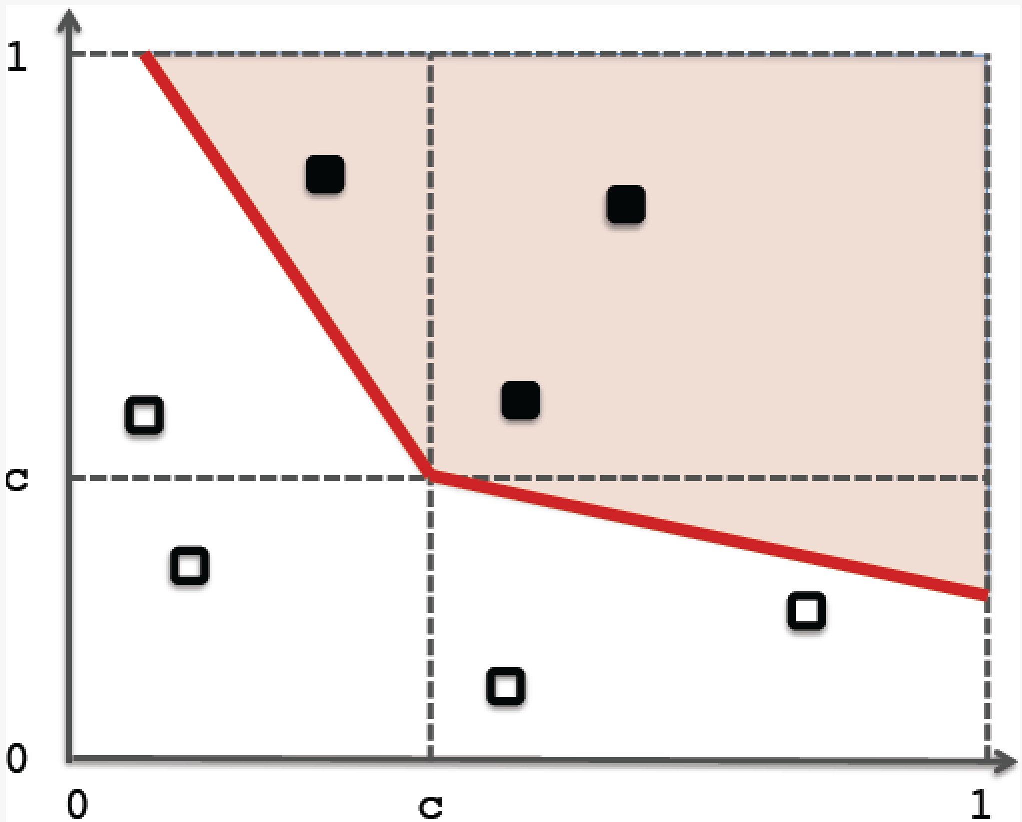
- Max – maximum positive interaction (synergy)



$$U(a) = max\{f_1(a), f_2(a)\} \geq c, \quad \mu(\{f_1\}) = 1, \ \mu(\{f_2\}) = 1, \ \mu(\{f_1, f_2\}) = 1$$

Isoquants of the Choquet integral for two criteria – special cases

- 2-additive Choquet – positive interaction (synergy)

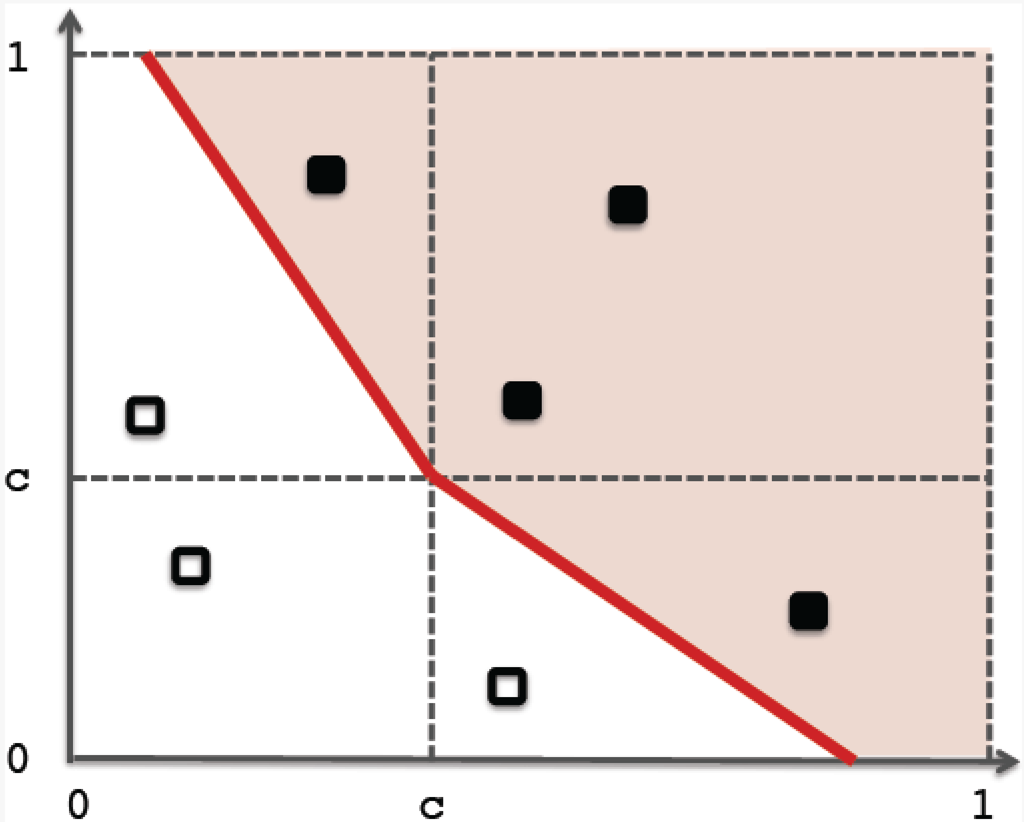


$$U(a) = \mu(\{f_1\})\, f_1(a) + \mu(\{f_2\})\, f_2(a) + [\mu(\{f_1, f_2\}) - \mu(\{f_1\}) - \mu(\{f_2\})]\, min\{f_1(a), f_2(a)\} \geq c$$

positive interaction when $\mu(\{f_1, f_2\}) > \mu(\{f_1\}) + \mu(\{f_2\})$

Isoquants of the Choquet integral for two criteria – special cases

- 2-additive Choquet – positive interaction (synergy)



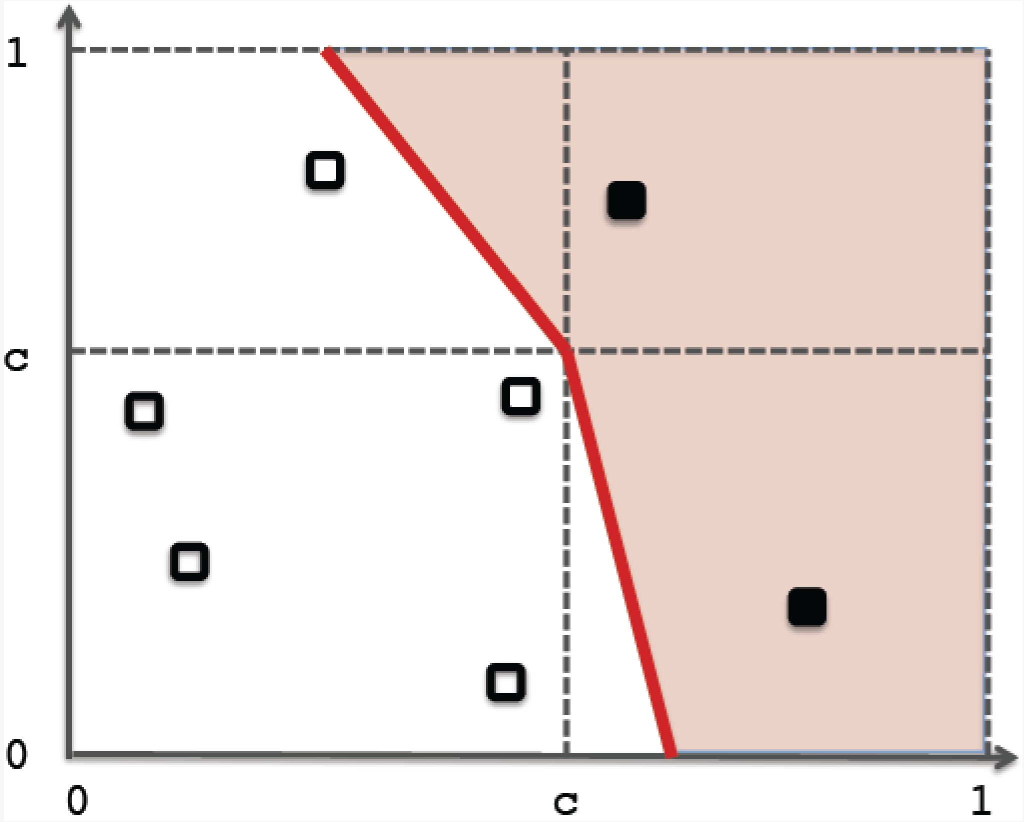
greater capacity = weight of f_1 than before

$$U(a) = \mu(\{f_1\})\, f_1(a) + \mu(\{f_2\})\, f_2(a) + [\mu(\{f_1, f_2\}) - \mu(\{f_1\}) - \mu(\{f_2\})]\, min\{f_1(a), f_2(a)\} \geq c$$

positive interaction when $\mu(\{f_1, f_2\}) > \mu(\{f_1\}) + \mu(\{f_2\})$

Isoquants of the Choquet integral for two criteria – special cases

- 2-additive Choquet – negative interaction (redundancy)



$$U(a) = \mu(\{f_1\})\, f_1(a) + \mu(\{f_2\})\, f_2(a) + [\mu(\{f_1, f_2\}) - \mu(\{f_1\}) - \mu(\{f_2\})]\, min\{f_1(a), f_2(a)\} \geq c$$

negative interaction when $\mu(\{f_1, f_2\}) < \mu(\{f_1\}) + \mu(\{f_2\})$

NEMO-II-Ch main points

- Start with the linear value function as preference model
- Ask every q iterations DM's preferences by comparing two non-dominated solutions
- Order the solutions by checking if there exists at least one compatible model for which x is preferred to all other solutions
- Within the same front order the solutions with respect to the crowding distance
- Switch to the 2-additive Choquet integral preference model as soon as the linear model is not able to represent the preferences of the DM anymore

Checking if there exists a model compatible with the DM's preferences for which *x* is preferred to all other solutions

$$\max \varepsilon, \quad \text{s.t.}$$

$$Ch_\mu\left(w_1 f_1(b),\ldots,w_n f_n(b)\right) - Ch_\mu\left(w_1 f_1(a),\ldots,w_n f_n(a)\right) + \varepsilon \le 0, \ \text{if } a \succ b$$

$$Ch_\mu\left(w_1 f_1(y),\ldots,w_n f_n(y)\right) - Ch_\mu\left(w_1 f_1(x),\ldots,w_n f_n(x)\right) + \varepsilon \le 0, \ \forall y \in A \setminus \{x\}$$

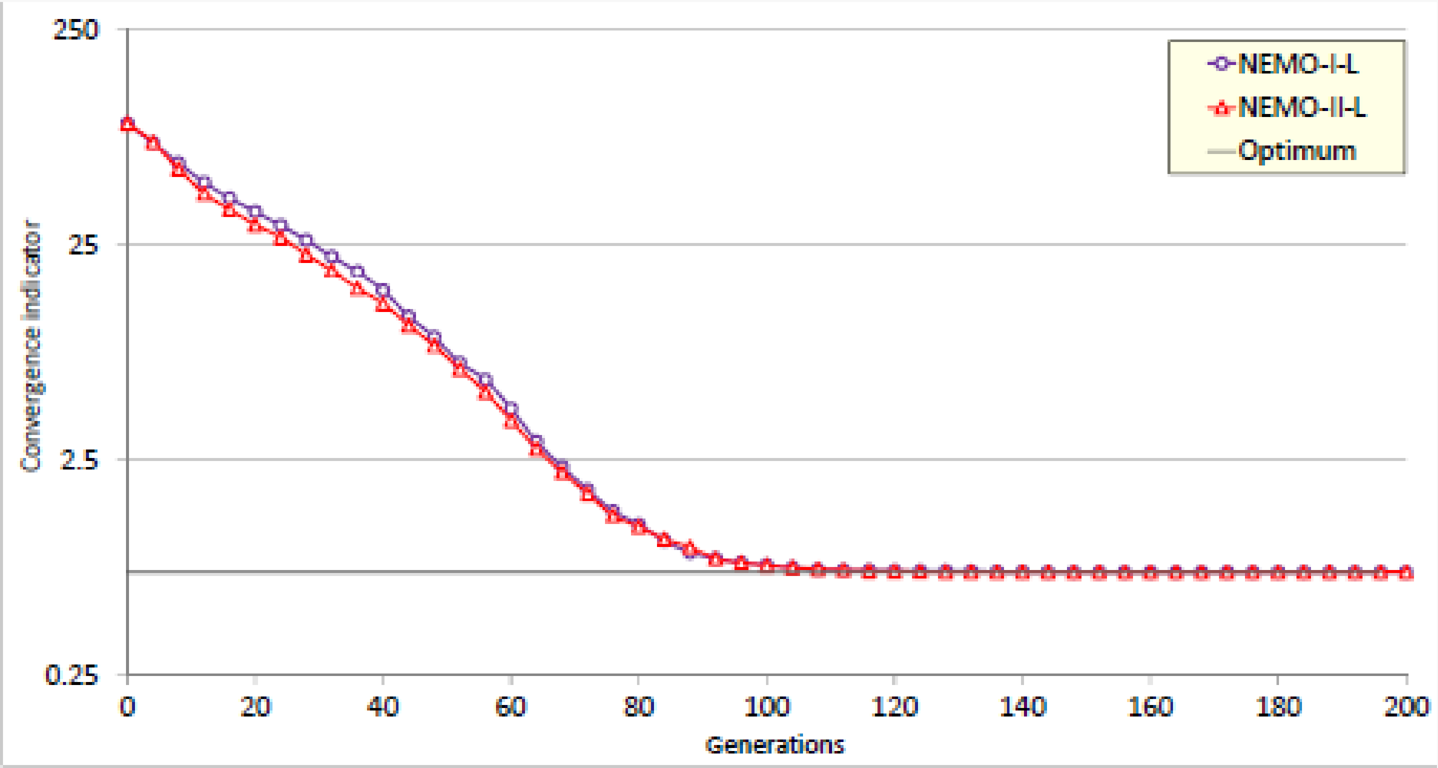
$$\sum_{i=1}^{n} w_i = 1$$

$$m(\emptyset) = 0, \quad \sum_{i \in N} m(\{i\}) + \sum_{\{i,j\} \subseteq N} m(\{i,j\}) = 1, \quad m(\{i\}) \ge 0, \quad \forall i \in N$$

$$m(\{i\}) + \sum_{j \in T} m(\{i,j\}) \ge 0, \ \forall i \in N \text{ and } \forall T \subseteq N \setminus \{i\}, \ T \ne \emptyset$$

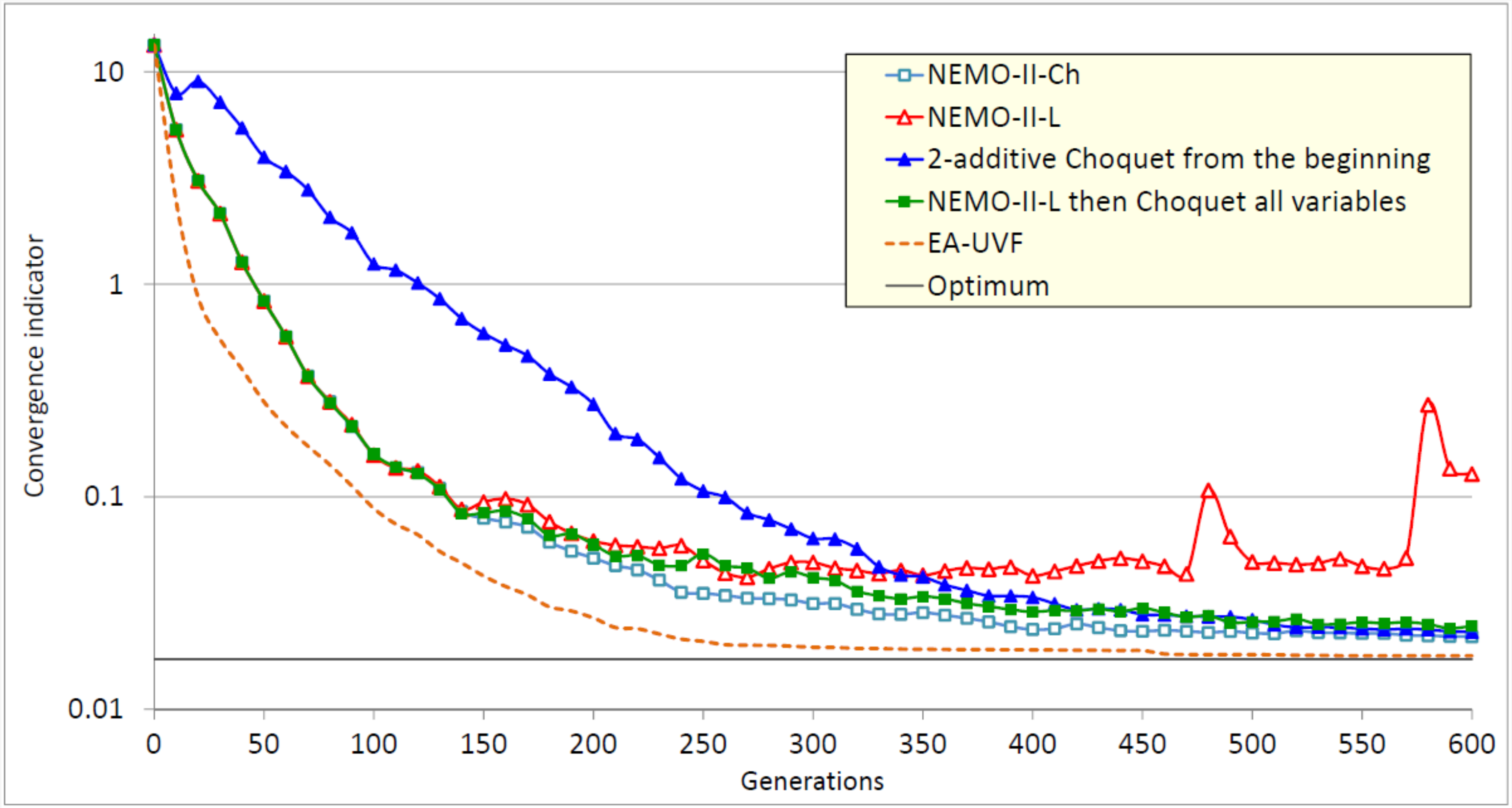
- First, we consider the set of weights $(w'_1,\ldots,w'_n)$ such that $w'_1 f_1(x) = \ldots = w'_n f_n(x)$
- If there is not any compatible capacity then we use the **Nelder-Mead** method [Nelder & Mead 1965]

Comparing NEMO-I-L and NEMO-II-L (ZDT1-2D)



- Complexity: O(n) instead of O(n^2)
- Similar convergence

Why NEMO-II-Ch? (DTLZ1-5D)



- DM compares two n-d solutions in the same front every 10 iterations
- It is better to start with the simplest model (the linear one);
- Passing to the 2-additive Choquet integral preference model produces better results than passing to the complete Choquet integral model;
- In NEMO-II-Ch interactions between <u>pairs</u> of criteria are considered.

Experimental setup

- ZDT1 and ZDT2 test functions on 2D;
- DTLZ1 and DLTZ2 test functions on 3D and 5D;
- User preferences according to Chebyshev with different weights;
- Comparisons with
  - NSGA-II,
  - NEMO-II-L,
  - NEMO-II-PL2,
  - NEMO-II-Ch,
  - EA-UVF.
- Results averaged over 50 replications.

ZDT2

$$f_1 = x_1$$

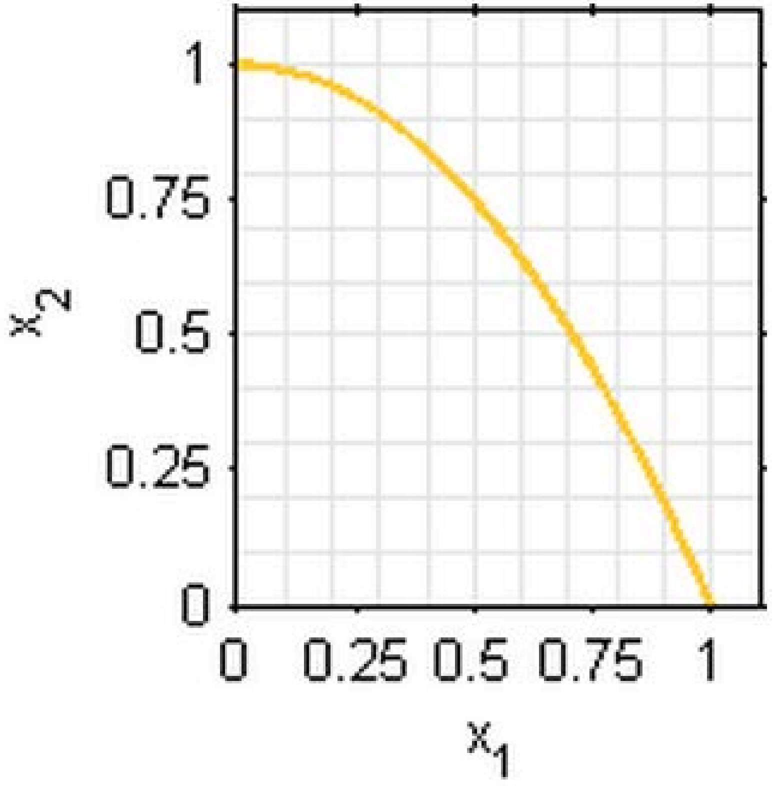
$$f_2 = g(\vec{x}) \cdot \left[1.0 - (x_1/g(\vec{x}))^2\right]$$

$$g(\vec{x}) = 1 + \frac{9}{n-1}\left(\sum_{i=2}^{n} x_i\right)$$

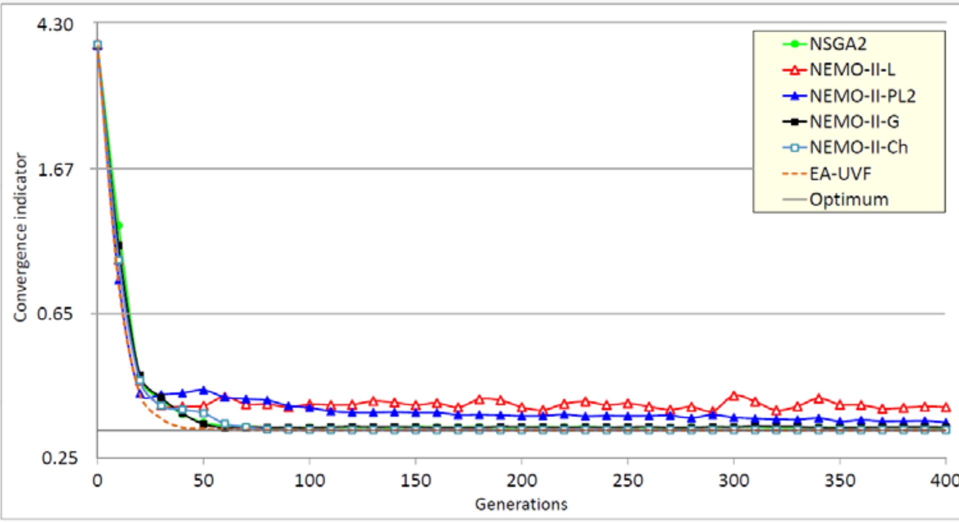
$$0 \leq x_i \leq 1, i = 1, \ldots, n$$

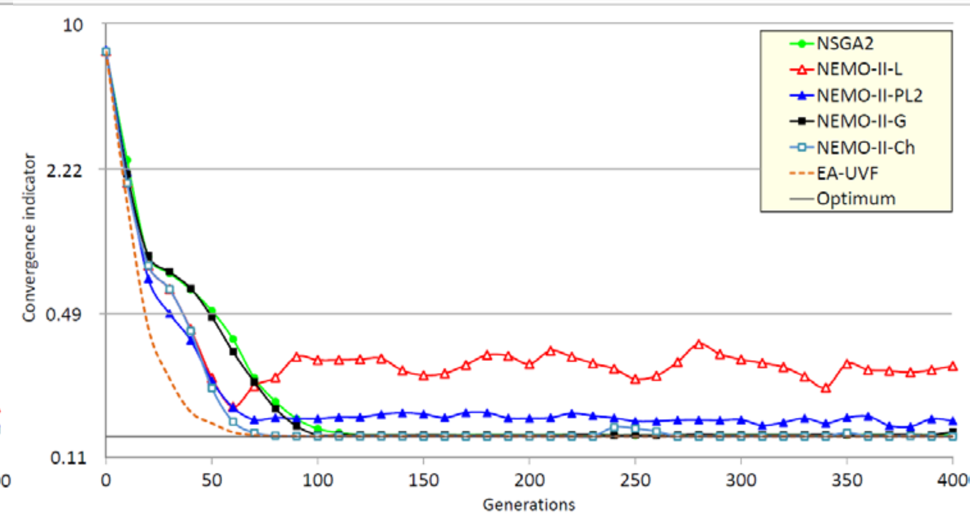
Pareto front

$$x_2 = 1 - x_1^2$$



Results on ZDT2





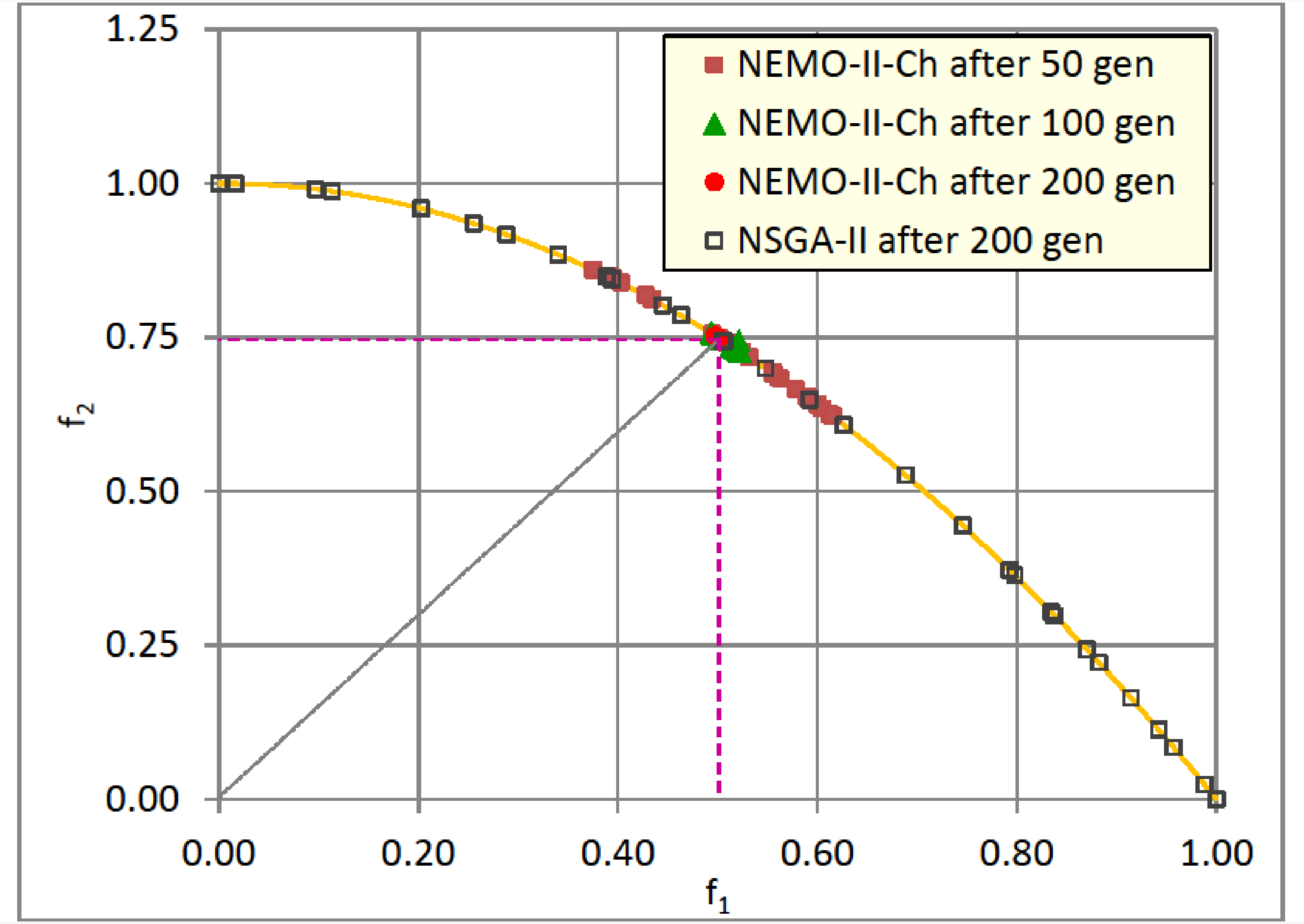
	ZDT2	
	middle	extreme
NSGA-II	199.96 ± 2.02	255.08 ± 4.67
NEMO-II-L	221.24 ± 2.31	252.23 ± 3.47
NEMO-II-PL2	205.17 ± 1.40	254.36 ± 3.79
NEMO-II-G	199.99 ± 1.08	252.38 ± 2.48
NEMO-II-Ch	**189.55** ± 1.61	**225.27** ± 4.56

	w_1	w_2
ZDT2-2D Cheb. (middle)	0.6	0.4
ZDT2-2D Cheb. (extreme)	0.15	0.85

- NEMO-II-L does not converge to the correct point
- NSGA-II and NEMO-II-G converge more slowly than NEMO-II-Ch

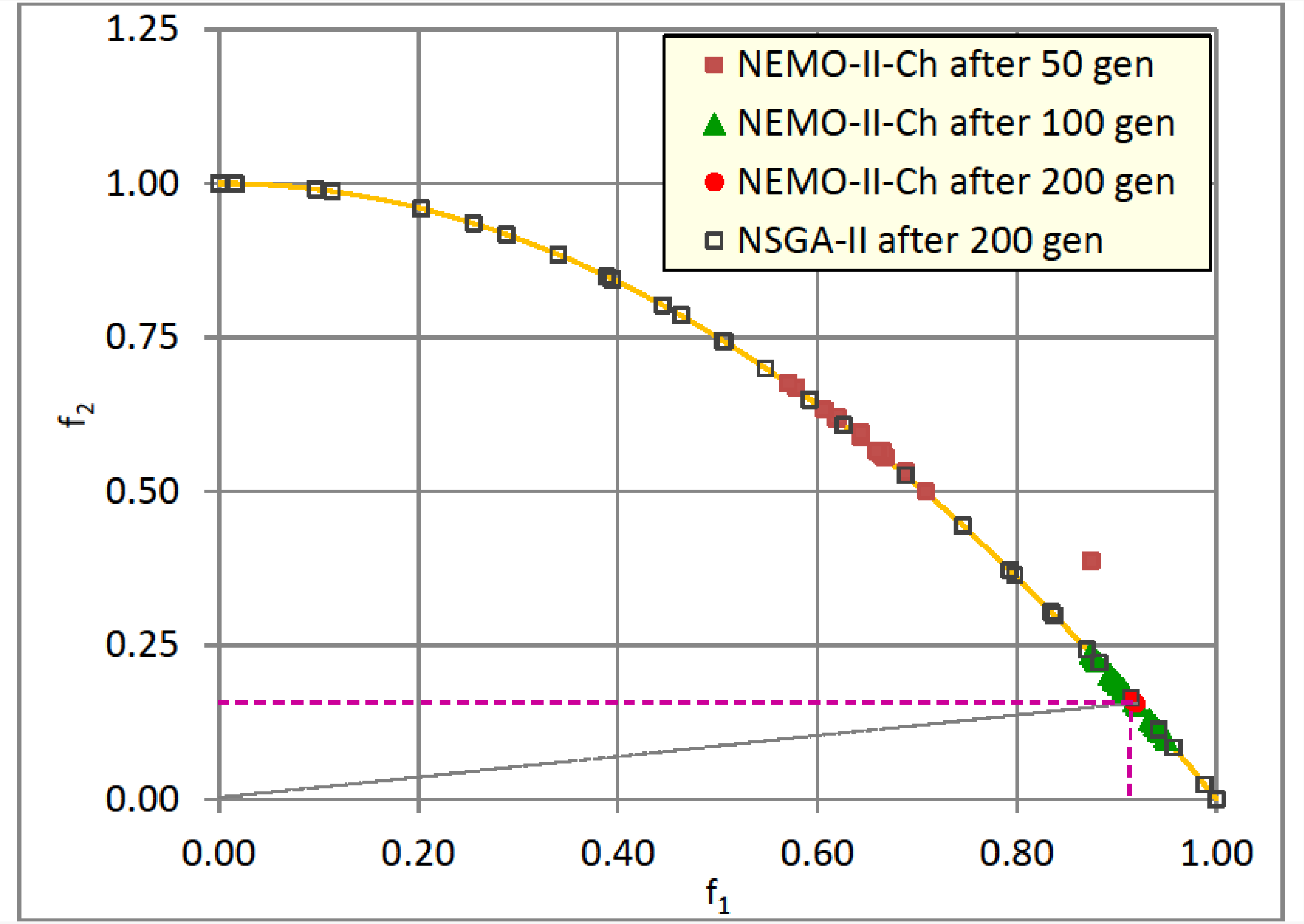
ZDT2

Value function of an artificial user: weighted Chebyshev metric



ZDT2

Value function of an artificial user:
weighted Chebyshev metric



DTLZ1-3D

Minimize $F = (f_1(\vec{x}), f_2(\vec{x}), f_3(\vec{x}))$, where

$$f_1(\vec{x}) = \frac{1}{2} x_1 x_2 (1 + g(\vec{x})),$$

$$f_2(\vec{x}) = \frac{1}{2} x_1 (1 - x_2)(1 + g(\vec{x})),$$

$$f_2(\vec{x}) = \frac{1}{2} (1 - x_1)(1 + g(\vec{x})).$$

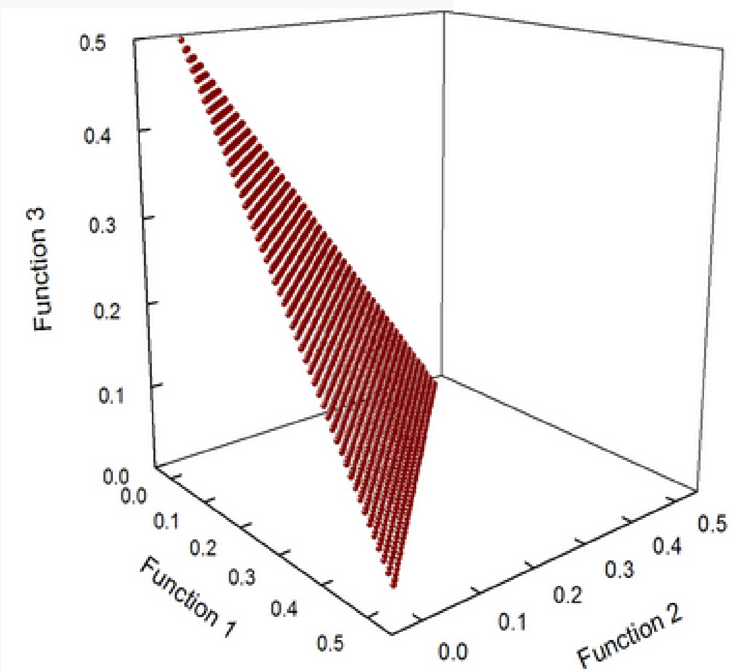
and

$$g(\vec{x}) = 100[10 + \sum_{i=3}^{n} (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))].$$

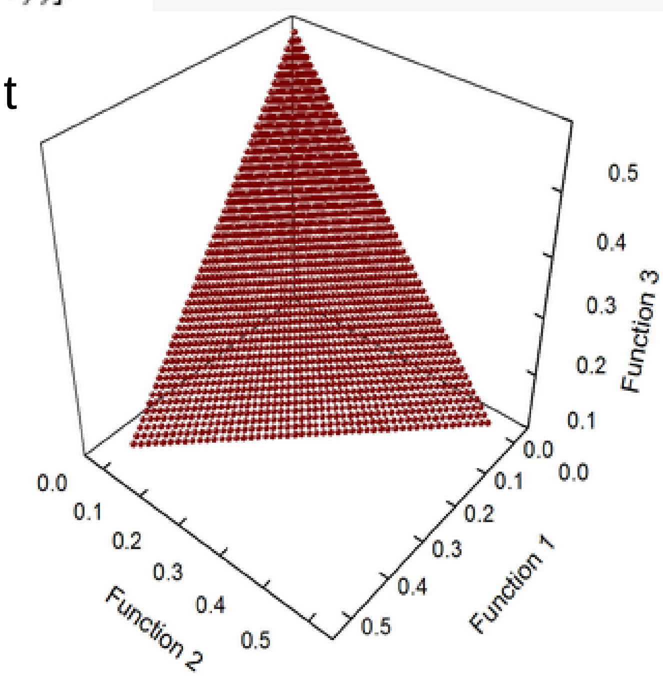
$n = 12,$

$0 \leq x_i \leq 1,$

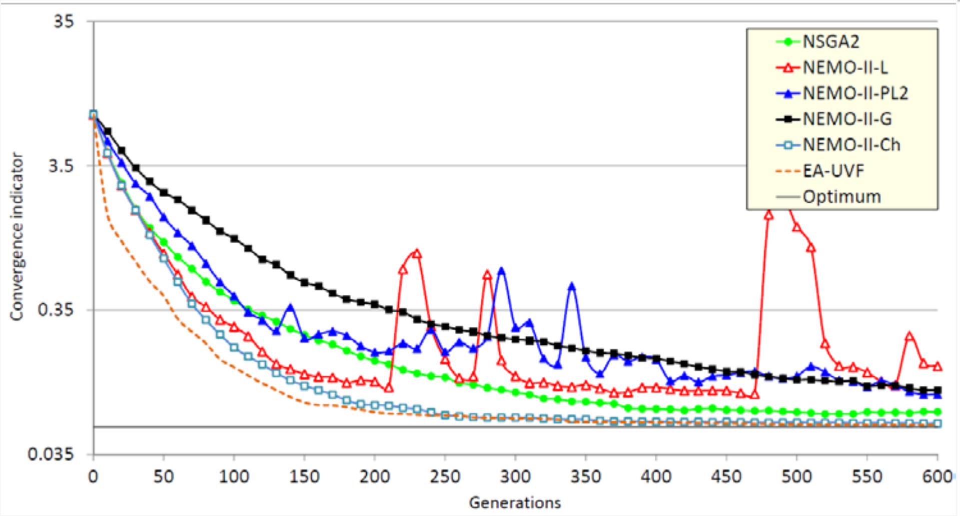
$i = 1, \ldots, 12$

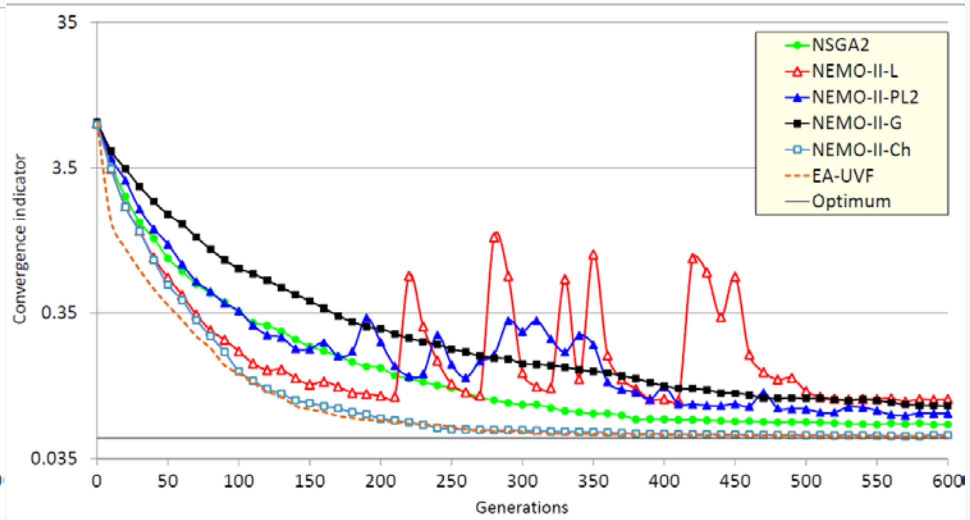


Pareto front



Results on DTLZ1-3D



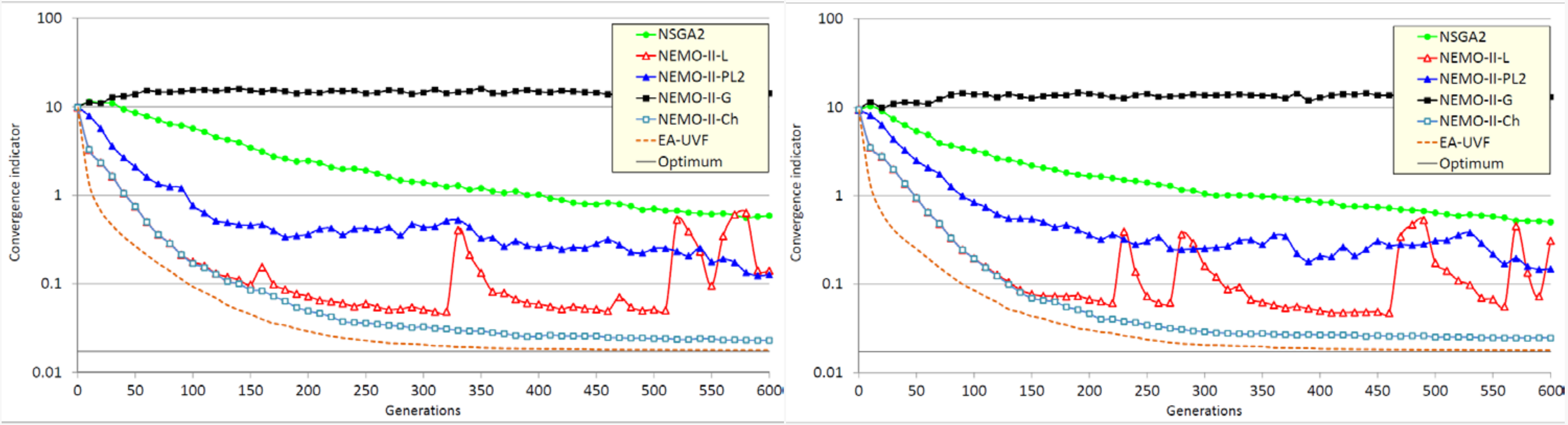


	DTLZ1	
	middle	extreme
NSGA-II	334.78± 9.64	291.66 ± 7.93
NEMO-II-L	457.72 ± 16.57	395.34 ± 11.53
NEMO-II-PL2	446.30±12.74	361.94 ± 9.14
NEMO-II-G	562.98 ± 13.34	439.64 ± 10.40
NEMO-II-Ch	**301.49** ± 11.56	**260.68** ± 9.96

	w_1	w_2	w_3
DTLZ1-3D Cheb. (middle)	0.3	0.4	0.3
DTLZ1-3D Cheb. (extreme)	0.2	0.3	0.5

- NEMO-II-Ch is able to get very close to the correct point
- NSGA-II converges significantly slower
- NEMO-II-L and NEMO-II-PL2 show erratic behaviour

Results on DTLZ1-5D



	w_1	w_2	w_3	w_4	w_5
DTLZ1-5D Cheb. (extreme 1)	0.1	0.15	0.2	0.25	0.3
DTLZ1-5D Cheb. (extreme 2)	0.3	0.25	0.2	0.15	0.1

	DTLZ1	
	extreme 1	extreme 2
NSGA-II	1746.86 ± 70.29	548.00 ± 50.35
NEMO-II-L	560.19 ± 30.33	218.49 ± 40.64
NEMO-II-PL2	586.27 ± 22.49	299.10 ± 21.89
NEMO-II-G	8569.95 ± 40.16	7531.91 ± 38.14
NEMO-II-Ch	**202.53 ± 9.62**	**204.83 ± 8.39**

- NEMO-II-Ch obtains much better results than any of the other methods
- NSGA-II performs quite poorly
- NEMO-II-L and NEMO-II-PL2 show again a very erratic behaviour

Conclusions

- Preference model needs to be chosen carefully
  - Too simple -> unable to capture user’s preferences
  - Too flexible -> unable to generalize, slow to learn
  - Idea: start simple, increase complexity once unable to capture user’s preferences
- Various models are used in literature, often without deep reflexion
- NEMO framework
  - NEMO-0: learn a single preference function -> complete order based on a sensible idea
  - NEMO-I: pairwise necessary preference relations
  - NEMO-II: compatible value function that prefers a given solution
- Choquet integral is a useful preference model, able to capture interactions
- NEMO-II-Ch is an adaptive interactive EMO algorithm

Discussion

* This work has been partly funded by the Programma Operativo Nazionale Ricerca & Competitività 2007-2013 within the project PON04a2 E SINERGREEN-RES-NOVAE