MCDA81, Annecy, 26-28 March 2015

Exploring the robustness of elicited weights in MCDA approaches by using new measures and feedbacks

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Structure

- Introduction
- Robustness measures
- Support the Decision Maker to understand better
- Feedbacks for more robust preference model
- Conclusions – Perspectives

European Union
European Social Fund
OPERATIONAL PROGRAMME EDUCATION AND LIFELONG LEARNING investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS
MANAGING AUTHORITY
Co-financed by Greece and the European Union
NSRF 2007-2013 programme for development
EUROPEAN SOCIAL FUND

This research has been co-financed by the European Union (European Social Fund) and Greek national funds through the Operational Program "Education and Lifelong Learning"

Robust MCDA

The problem

The assessment of criteria weights in multicriteria methods of value systems, such as disaggregation - aggregation approach often leads to the elicitation of preference models with low degree of robustness. Namely, several weight vectors, which are compatible with the DM's preference structure, are estimated. In many cases, wide range of the weight of each criterion is observed, as well as, several rank reversals of the criteria importance in the different weight vectors.

Given the fact that the DM shall be aware of such phenomena of low robustness, so that (s)he can be protected when applying the estimated preference model, the key point of this research is the development of a methodological approach which will provide the framework to measure the level of robustness of the estimated preference model and facilitate the exploration of its nature.

What - How

This research work presents a methodological frame which focus on three main issues:

a) the evaluation of the degree of robustness of the elicited weights,

b) the provision of support to the DM towards the exploration of the nature of the probable low robustness and the deeper understanding of his/her preferential structures and

c) the estimation of more robust preference models by applying a set of feedbacks.

Robustness Analysis in UTA methods

The UTA Methods of Multicriteria disaggregation - aggregation approach for discrete alternative actions lead to the estimation of DMs' additive value preference:

$$[\min] F = \sum_{m=1}^{k} \left(\sigma^{+}(a_m) + \sigma^{-}(a_m) \right) \quad \text{subject to:}$$

$$HP1 \begin{cases} \left. \begin{array}{l} \sum_{i=1}^{n} p_i u_i \left[g_i(a_m) \right] - \sigma^{+}(a_m) + \sigma^{-}(a_m) - \sum_{i=1}^{n} p_i u_i \left[g_i(a_{m+1}) \right] - \sigma^{+}(a_{m+1}) + \sigma^{-}(a_{m+1}) \geq \delta \text{ if } a_m \mathrm{P} a_{m+1} \\ \sum_{i=1}^{n} p_i u_i \left[g_i(a_m) \right] - \sigma^{+}(a_m) + \sigma^{-}(a_m) - \sum_{i=1}^{n} p_i u_i \left[g_i(a_{m+1}) \right] - \sigma^{+}(a_{m+1}) + \sigma^{-}(a_{m+1}) = 0 \text{ if } a_m \mathrm{I} a_{m+1} \end{array} \right\} \forall m \\ \sum_{i=1}^{n} p_i = 1, \text{ for } i = 1, 2, \ldots, n, \quad p_i \geq 0,\ \sigma^{+}(a_m) \geq 0,\ \sigma^{-}(a_m) \geq 0 \ \forall i \text{ and } m \end{cases}$$

The LPs of the post optimality analysis (heuristic approach) may have the following form:

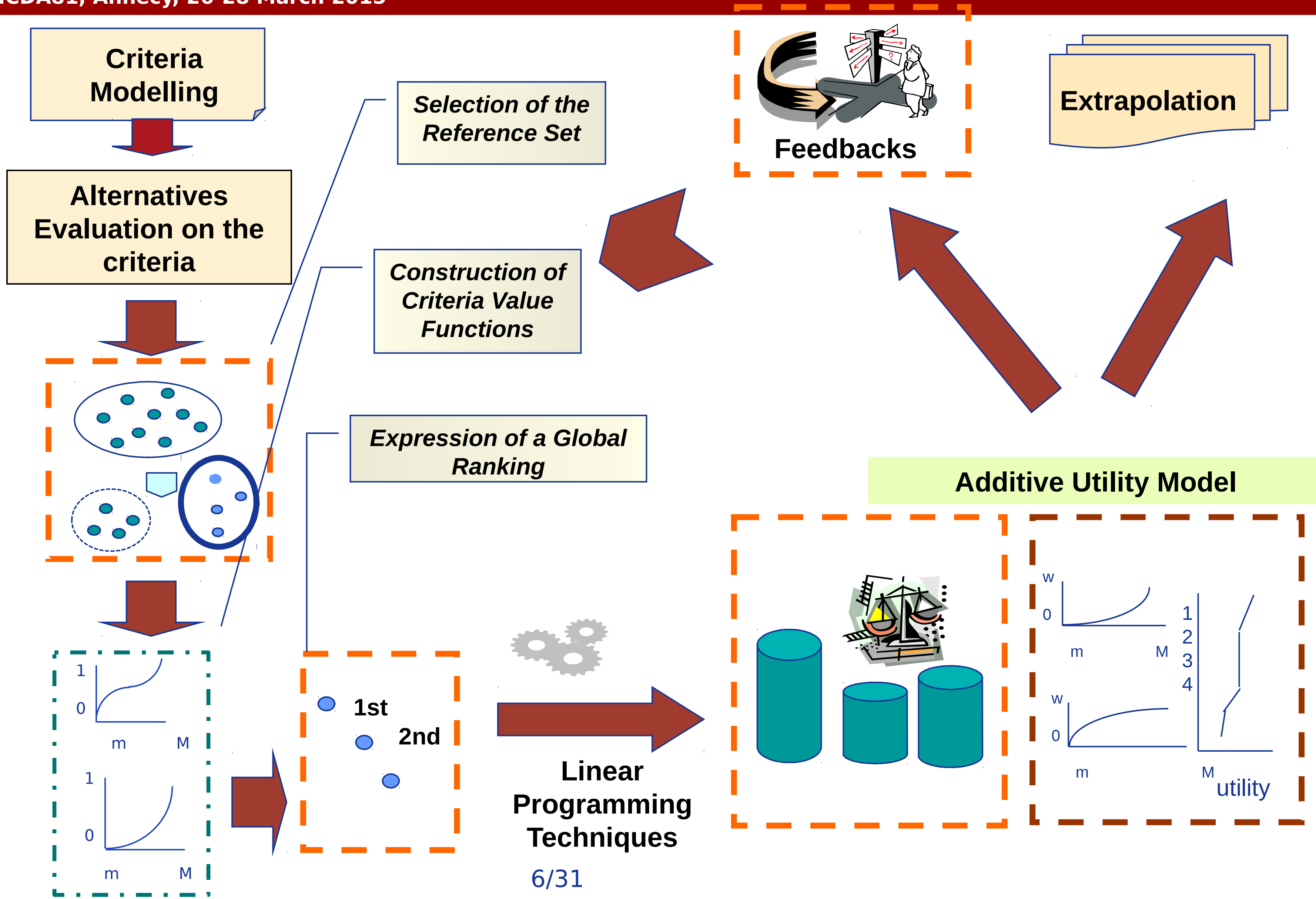
$$[\min] \ or \ [\max] F_i = p_i ,\ i = 1, 2, \ldots, n$$

subject to:

$HP1$

$$\sum_{m=1}^{k} \left(\sigma^{+}(a_m) + \sigma^{-}(a_m) \right) = f^{*}$$

Steps of D-A approach (UTA II)



Solution of LP in UTA

The estimation of the parameters of the DM's value system can lead to:

A. Only one solution (Robust). There is only one vector of the weights.

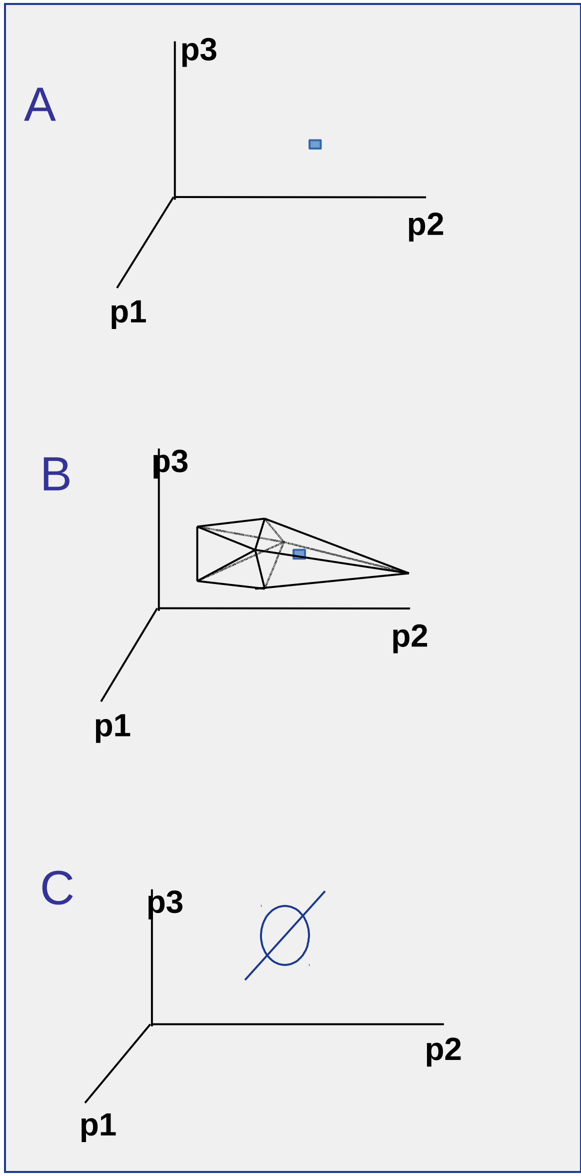
B. Infinite Solutions (Non Robust).

C. No Solutions, often in cases with extremely low structure.

Question?

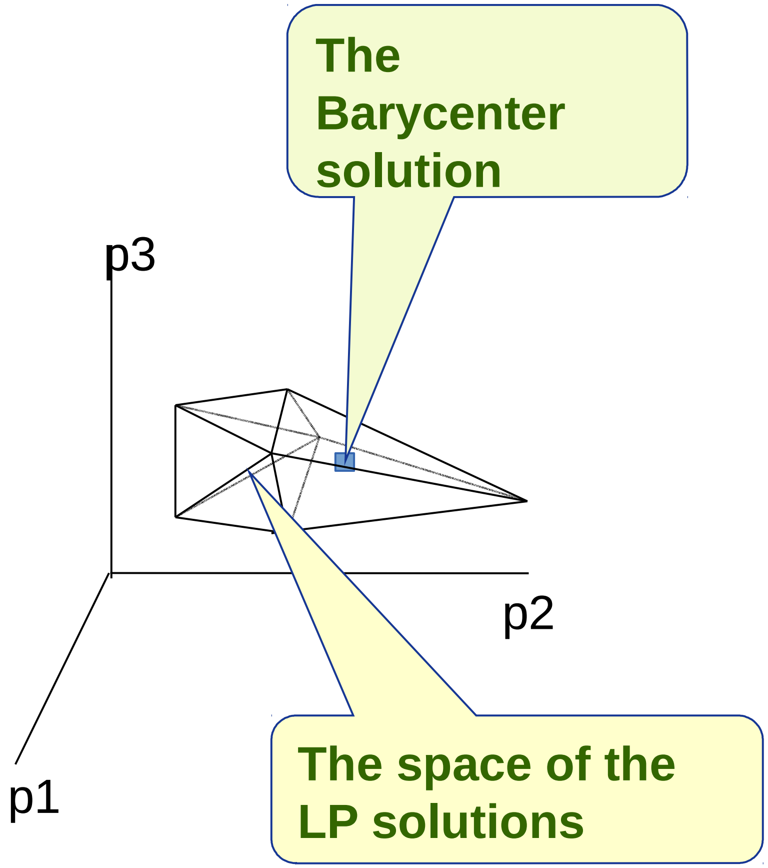
In non-robust cases which could be the best vector of weights to work with?

MINORA and MIIDAS systems (Siskos et al, 1993, 1999) utilise post optimal analysis solutions which are estimated by maximising the weight of every criterion. The mean solution (barycenter) constitute the working vector of weights



Hyper-polyhedron of post optimal solutions

Example of Hyper-Polyhedron of LP Solution for low robustness preference models and Barycenter for 3 criteria weights (p1, p2, p3)



Robustness Analysis in UTA methods

The presence of low robustness in the estimated preference model results to some crucial questions:

- Can a preference model with low robustness be accepted, while the criteria weights are falling into a wide range of values?
- Can a preference model be accepted, which presents reversal of criteria importance into the estimated hyper-polyhedron?
- Which is the degree of robustness that could be accepted for continuing the decision support process?

The need

Following a detailed evaluation and exploration of the robustness of the estimated preference model the DM will be in the position to decide whether:

- to accept the low robustness as a good represenattion of its preferences,

or

- to try to reduce it, by providing more information concerning the alternatives reference set, or/and the directly the criteria

At the latter case, new evaluation and exploration of the robustness will result to subsequent questions regarding the DM.

This process is accomplished in three major steps.

Robustness Measures

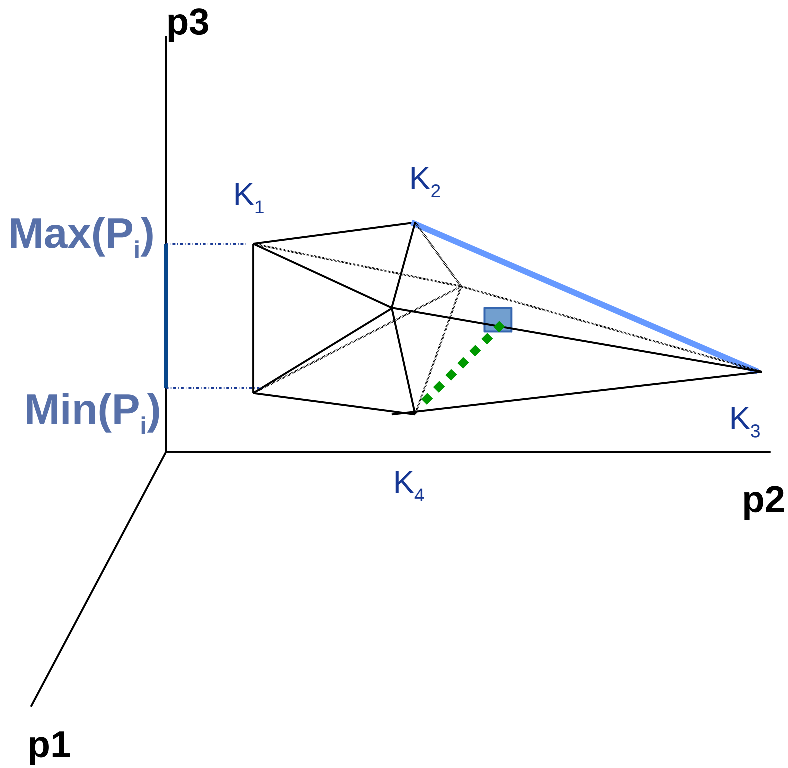
A. Minimum and maximum values of the criteria weights (Post Optimal Analysis)

$$\mu_i = \left(\max\left(p_{ij}\right) - \min\left(p_{ij}\right)\right),$$

p_{ij} *the weight of the i criterion in the j vertice,*

$i = 1,2,..,n, \qquad j = 1,2,..,m,$

n number of criteria and m number of vertices



B. The Average Stability Index (ASI)

$$ASI = 1 - \frac{\sum_{i=1}^{n} \sqrt{\left(m \left(\sum_{j=1}^{m} \left(p_{ij}\right)^2 \right) - \left(\sum_{j=1}^{m} p_{ij} \right)^2 \right)}}{m\sqrt{(n-1)}}$$

n= number of criteria

m = number of vertices of hyper-polyhedron

Robustness Measures

C. The infinitive set of solutions (hyper-polyhedron) resulting from the post optimality analysis in aggregation-disaggregation approach provides a lot of cases where we may observe rank reversal of the criteria weights.

A set of indices, Criterion Priorities Index (CPI_{ij}), is calculated for every pair of criteria, representing the degree of criterion weights reversal among the vertices of the hyper-polyhedron. CPIij is estimated with the following formulae:

$$CPI_{ij} = \frac{\#\left\{ p_i^k > p_j^k, \, for \; k = 1,\ldots,m \right\}}{m}, \quad for \, i,j = 1,2,..,n, i \neq j$$

where $(p_1, p_2, ..p_n)$ the vector at each vertex, n: number of criteria, m: number of vertices

$$CPI_{ij} + CPI_{ji} = 1 \;\; and \; CPI_{ij} \leq 1$$

$CPI_{ij} = 1$, when the criterion i has higher weight of criterion j for all the vertices of the hyper-polyhedron.

$CPI_{ij} = 0.5$, when the number of vertices with $p_i > p_j$ equals to the ones with $p_i \leq p_j$

Robustness Measures

The Priorities Reversal Index (PRI) is the normalized mean value of the CPI_{ij} indices and it is calculated with the formulae:

$$PRI = \frac{\sum_{i=1,\, j=i+1}^{n-1} \left| CPI_{ij} - 0.50 \right|}{\frac{n(n-1)}{2} 0.50}$$

PRI =0 when $CPI_{ij} = 0.5$ for all i, j=1,2,..n and i≠j, corresponding to the higher reversal of criteria priorities.

PRI=1 when $CPI_{ij} = 1$ or $CPI_{ji} = 0$ for all i, j = 1,2,...n and i≠j, corresponding to the absence of criteria priorities reversal on the vertices of the hyper-polyhedron.

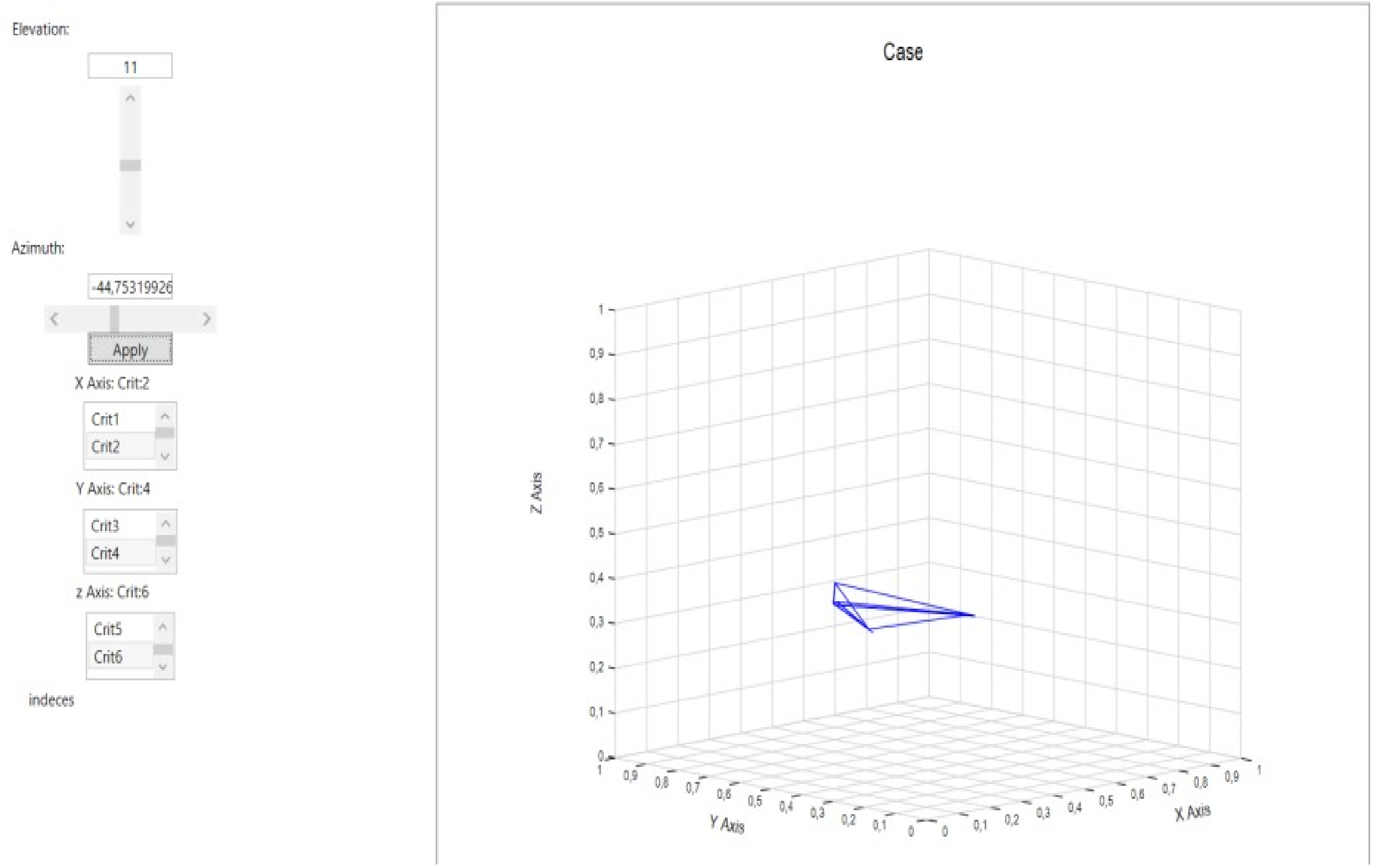
Explore the nature of the Low Robustness

The second step includes the exploration of the estimated preference model robustness, exploiting a set of tools including:

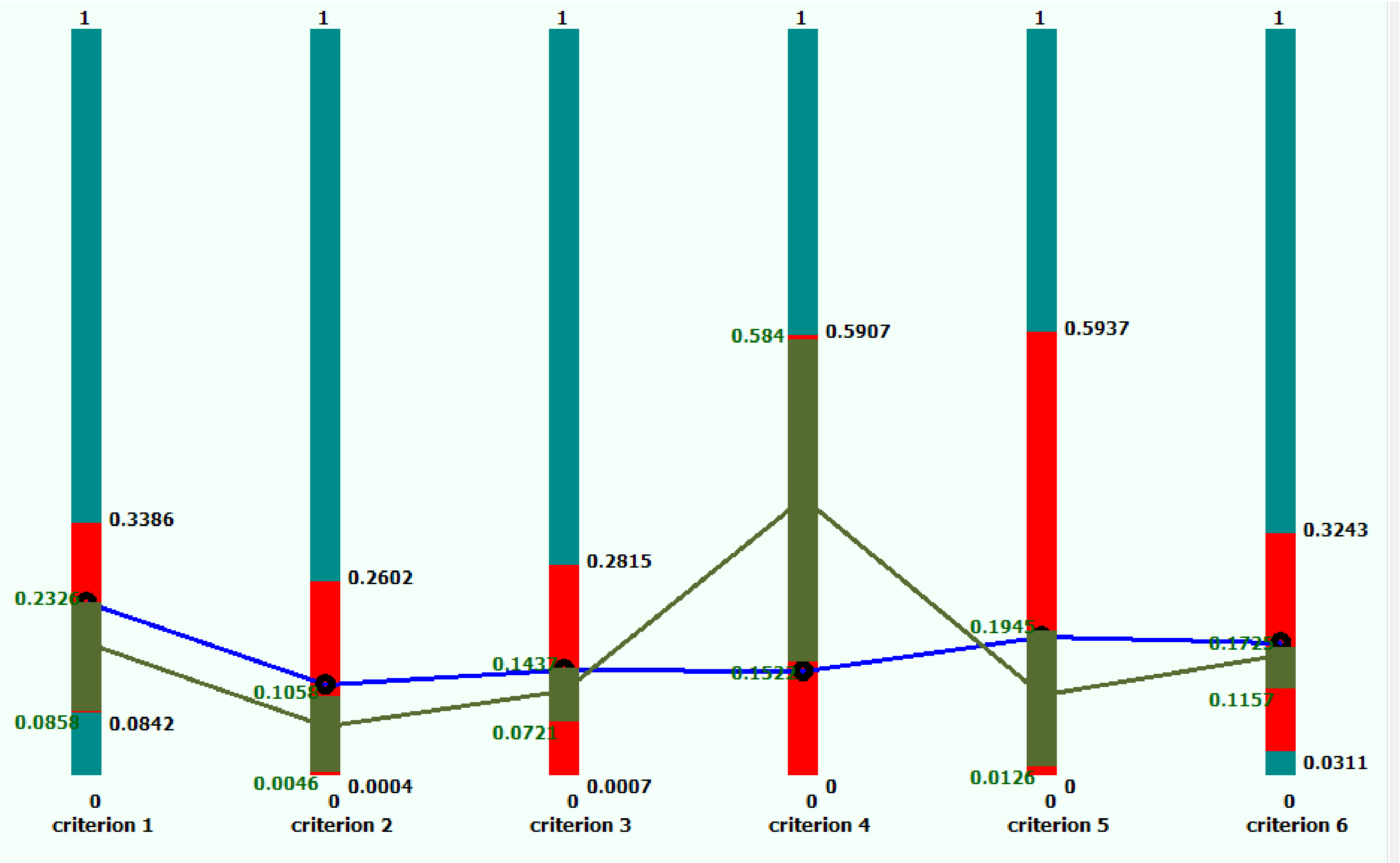
- visualisation of the hyper-polyhedron in 3-D graphical interface
- visual representations of weights ranges using of a parallel graph system
- a tomographical approach

Visualisation of the hyper-polyhedron

Visualisation of the hyper-polyhedron in 3-D graphical interface so as to provide the picture of the solution' hyper-space by selecting 3 dimensions every time:



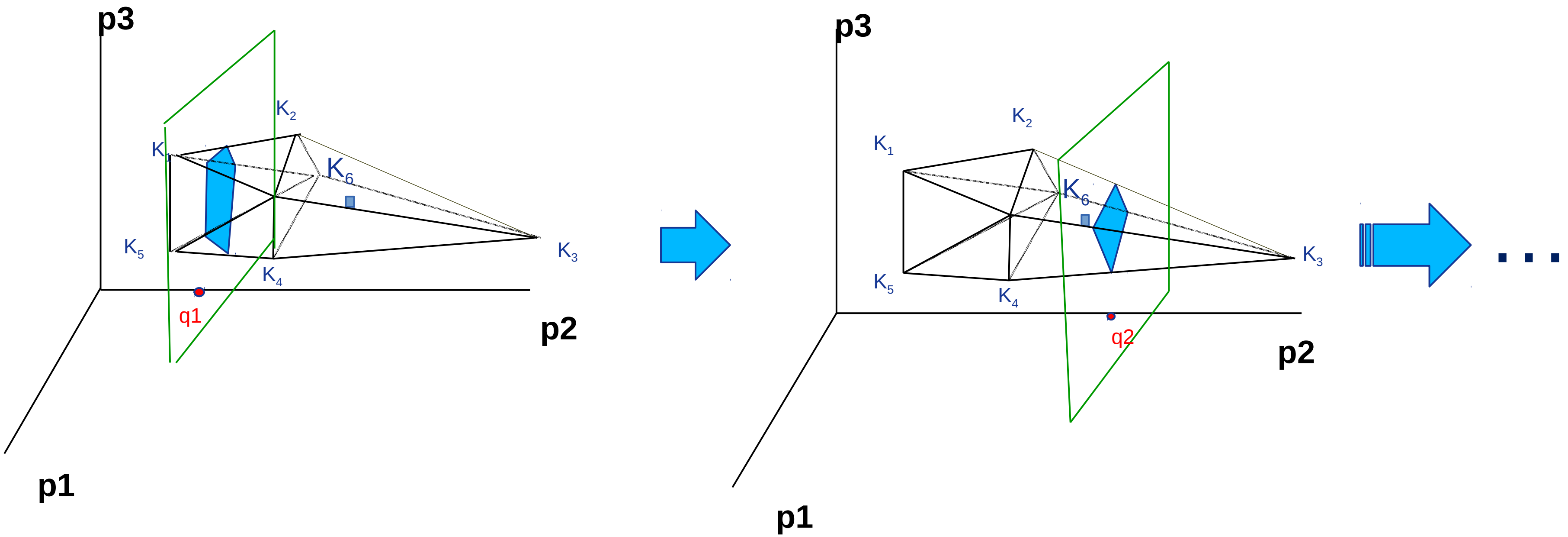
Exploitation of a parallel graph system, where the weights of the criteria are presented in bars in the scale of [0, 1]:



Tomographies of the Hyper-polyhedron

Tomographical approach constitutes a way to picture the degree of robustness into the hyper-polyhedron.

The idea is to discretize the n-dimensional estimated hyper-polyhedron of the criteria weights by using n-1 dimensional cutting hyper-polyhedra.



Tomographies of the Hyper-polyhedron

The module includes two different ways to proceed with tomographies:

a) to manually inspect the robustness of the hyper-polyhedron by selecting a criterion and a step. The tomographies are estimated starting from the minimum value of the criterion weight and at each iteration it is increased by the selected step.

b) to proceed with auto-running for all criteria with a selected step and calculate the indices of the robustness evaluation and present the results using a graph.

Tomographies of the Hyper-polyhedron

Criterion	Min()	BCenter	Max()	New Min	New Bce...	New Max	µi	New µi
Qualifications	0	0.0476520...	0.2254857...	0	0.0557117...	0.1500890...	0.2254...	0.1500...
Management	0	0.1095381...	0.2861218...	0.07	0.07	0.07	0.2861...	0
Decisions	0.2004964...	0.2603827...	0.3425688...	0.2216051...	0.2623493...	0.3209441...	0.1420...	0.0993...
Multiplicity	0.1821378...	0.2724103...	0.3513414...	0.2325851...	0.2812757...	0.3451694...	0.1692...	0.1125...
Results	0.1418190...	0.1785752...	0.2705225...	0.1439689...	0.1816929...	0.2466771...	0.1287...	0.1027...
FInancial	0	0.1314414...	0.2079738...	0.0776312...	0.1489702...	0.2020450...	0.2079...	0.1244...

Steps of Tomography

Criterion: Management

Step: 0.010

From: 0 To: 0.2861218764!

Current Value

0.07

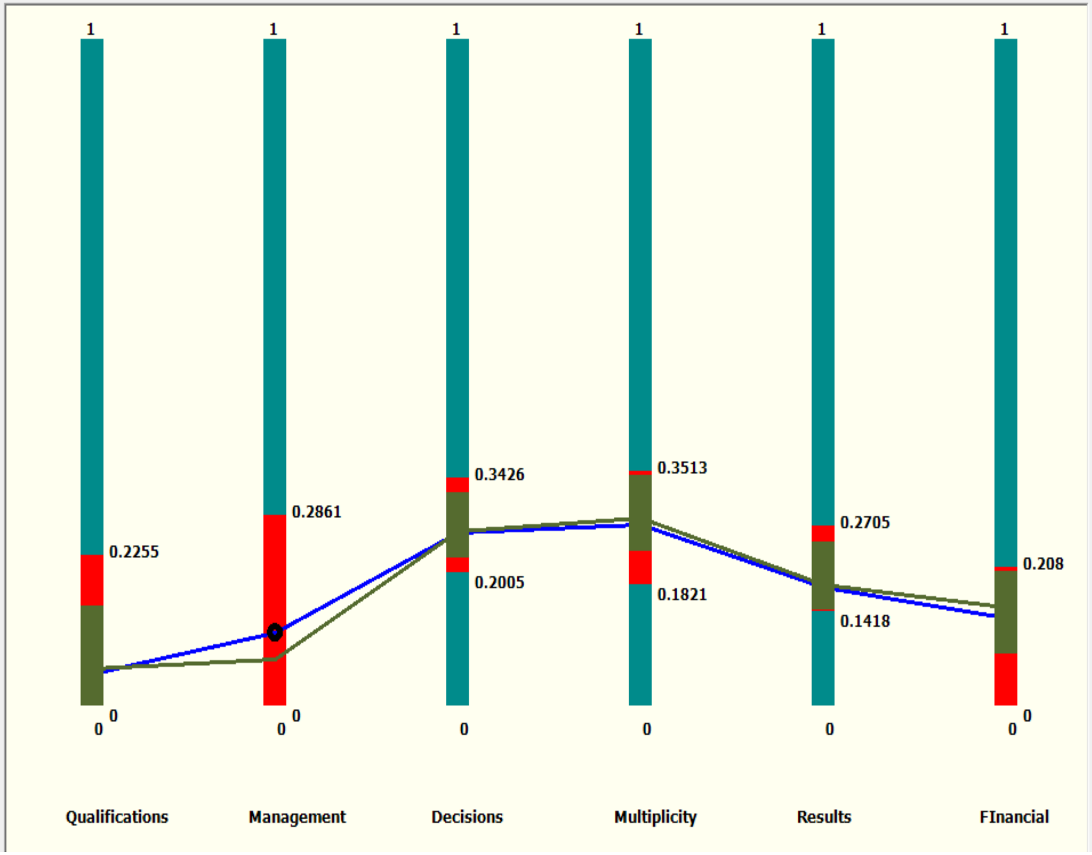
Next Step

Previous Step

Initialisation

Auto Tomography

Start Stop



Robustness Indices

Sinit	0.1933
Sdinit	0.2006
Snew	0.0982
Sdnew	0.1089
Snew/Sinit (%):	50.807
(1-Snew/Sinit)(%):	49.193
ASI Index:	0.9357
Initial ASI	0.904
Maximum ASI	0.9807

ASI Graph

Tomographies of the Hyper-polyhedron

Criterion	Min()	BCenter	Max()	New Min	New Bce...	New Max	µi	New µi
Qualifications	0	0.0476520...	0.2254857...				0.2254...	
Management	0	0.1095381...	0.2861218...				0.2861...	
Decisions	0.2004964...	0.2603827...	0.3425688...				0.1420...	
Multiplicity	0.1821378...	0.2724103...	0.3513414...				0.1692...	
Results	0.1418190...	0.1785752...	0.2705225...				0.1287...	
FInancial	0	0.1314414...	0.2079738...				0.2079...	

Steps of Tomography

Criterion:

Step: 0.001

From: To:

Current Value

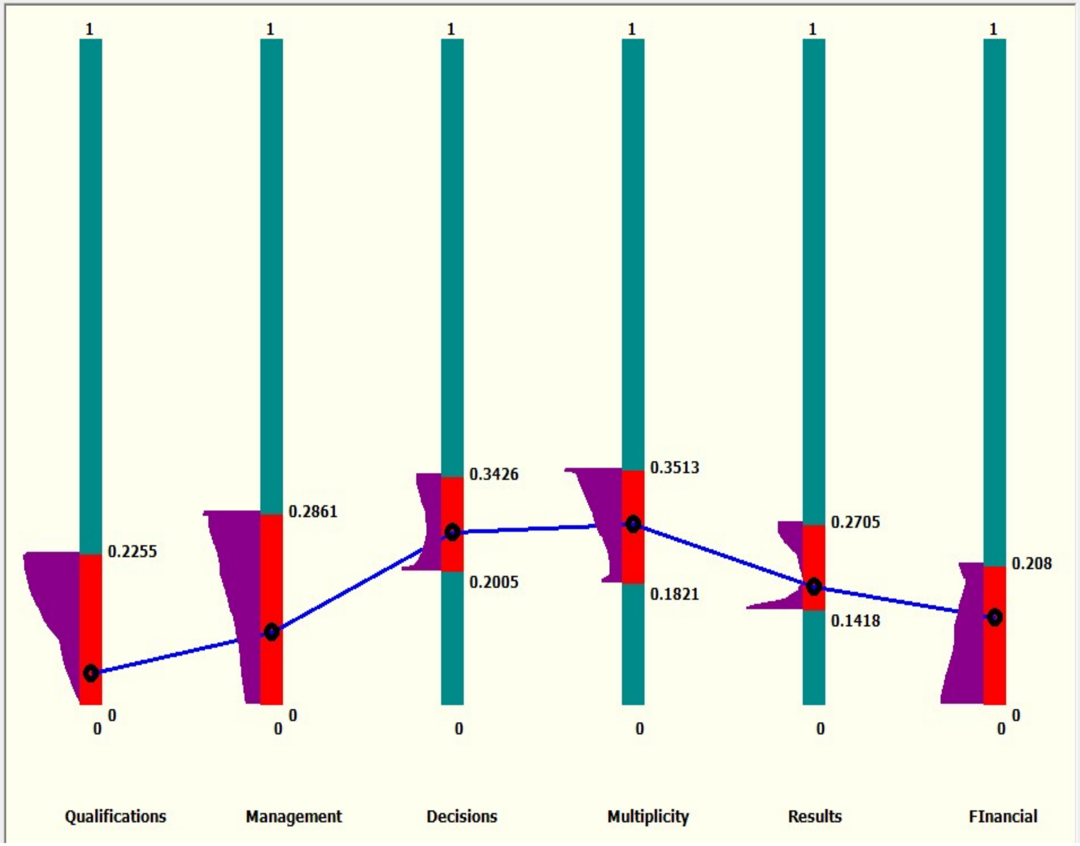
Next Step

Previous Step

Initialisation

Auto Tomography

Start Stop



Robustness Indices

Sinit	*0.1933*
Sdinit	*0.2006*
Snew	*0*
Sdnew	*0*
Snew/Sinit (%):	*0*
(1-Snew/Sinit)(%):	*100*
ASI Index:	*0.904*
Initial ASI	*0.904*
Maximum ASI	*0.9807*

ASI Graph

Tomographies of the Hyper-polyhedron

Criterion	Min()	BCenter	Max()	New Min	New Bce...	New Max	µi	New µi
ConstCost	0	0,0371649...	0,1634827...	0	0,0121815...	0,0487142...	0,1634...	0,0487...
MaintCost	0	0,0748165...	0,2636713...	0	0,0141810...	0,0591385...	0,2636...	0,0591...
Security	0,1282509...	0,1988838...	0,2659650...	0,2382509...	0,2382509...	0,2382509...	0,1377...	0
Aesthetic	0,1272882...	0,2235985...	0,3008351...	0,2334118...	0,2642033...	0,2837211...	0,1735...	0,0503...
Access	0	0,0425534...	0,0929259...	0	0,0345193...	0,0749993...	0,0929...	0,0749...
Reliab	0,3990445...	0,4229826...	0,4585249...	0,4163013...	0,4366637...	0,4560089...	0,0594...	0,0397...

Steps of Tomography

Criterion: Security

Step: 0,005

From: 0,1282509447 To: 0,2659650972

Current Value: 0,238251

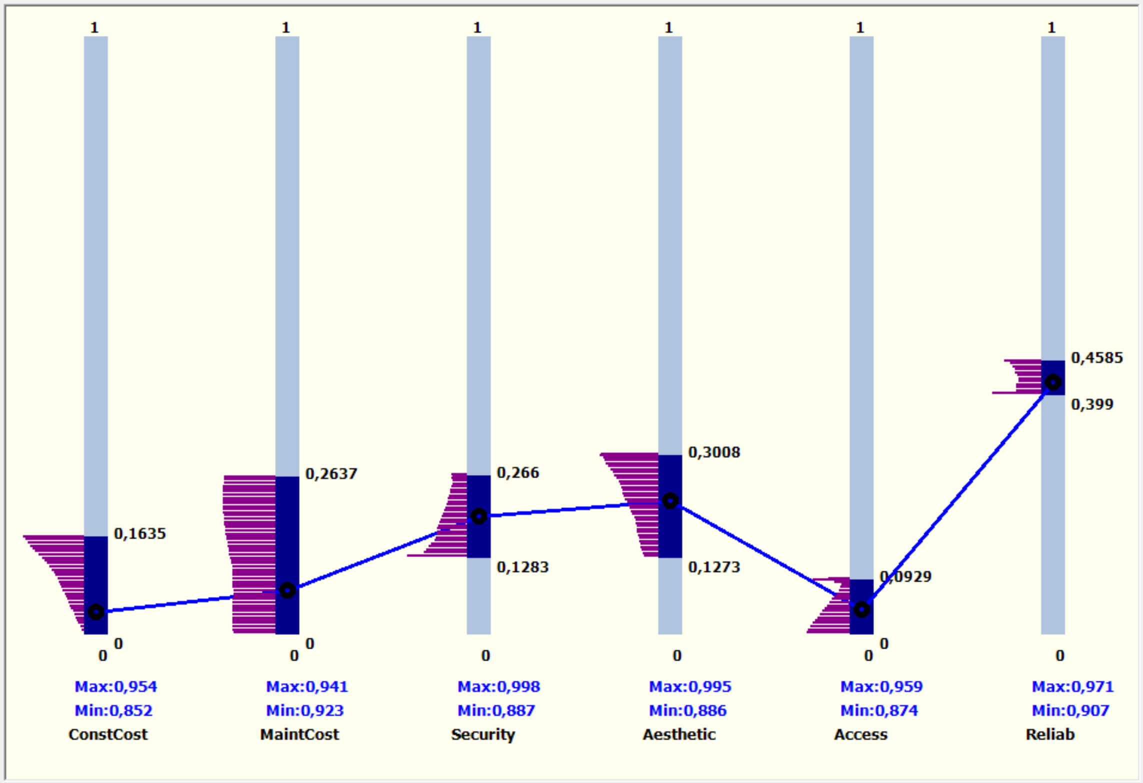
Next Step

Previous Step

Initialisation

Auto Tomography

Start Stop



ASI PR

Criteria	*Cr.1*	*Cr.2*	*Cr.3*	*Cr.4*	*Cr.5*	*Cr.6*
	Con...	*Mai...*	*Sec...*	*Aes...*	*Acc...*	*Reli...*
ConstCost	*0*	*0,32*	*0,02*	*0*	*0,42*	*0*
MaintCost	*0,66*	*0*	*0,17*	*0,16*	*0,67*	*0*
Security	*0,98*	*0,83*	*0*	*0,02*	*1*	*0*
Aesthetic	*1*	*0,84*	*0,98*	*0*	*1*	*0*
Access	*0,56*	*0,32*	*0*	*0*	*0*	*0*
Reliab	*1*	*1*	*1*	*1*	*1*	*0*

PRI: 0,8079

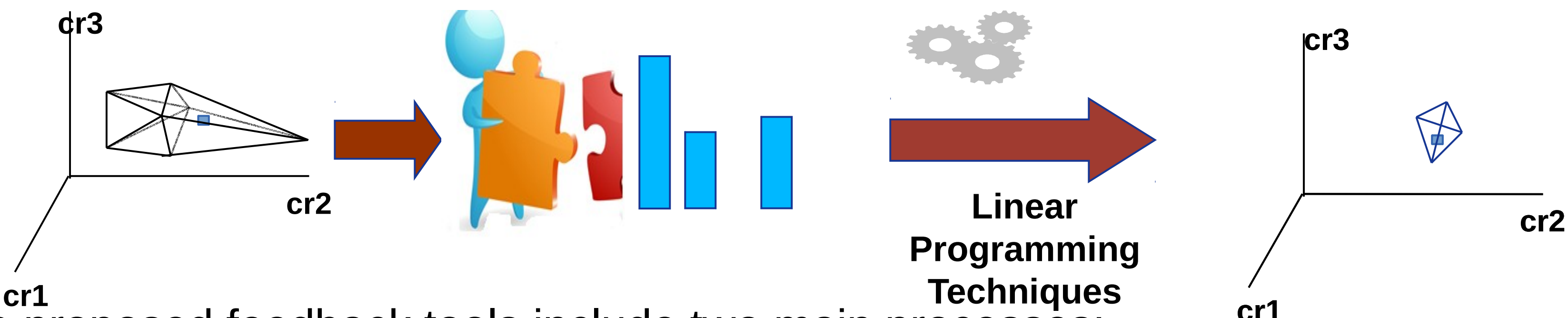
No of Tomographies: 186

ASI Graph

Feedbacks to increase Robustness

The estimation of preference models with low robustness is something frequently observed and probably undesired. Nevertheless, it can be considered as the starting point for new dialogues with the DM in order to receive additional preference information, which may lead to the revision of the preference model towards an acceptable one.

This additional information will impose additional constraints to the current linear program and probably lead to the estimation of a more robust preference model through.



The proposed feedback tools include two main processes:

- shrinking the hyper-polyhedron
- providing specific pairwise priorities on selected criteria

Shrinking the hyper-polyhedron

Criteria	Min()	BC	MaxV	New Min	New BC	New Max	µi	µi (new)
Qualifications	0	0.0476520...	0.2254857...	0.1	0.1	0.1	0.2254...	0
Management	0	0.1095381...	0.2861218...	0.1	0.1	0.1	0.2861...	2.7755...
Decisions	0.2004964...	0.2603827...	0.3425688...	0.2904964...	0.2904...	0.2904964...	0.1420...	0
Multiplicity	0.1821378...	0.2724103...	0.3513414...	0.2641187...	0.2641...	0.2641224...	0.1692...	3.6691...
Results	0.1418190...	0.1785752...	0.2705225...	0.1474073...	0.1474...	0.1474109...	0.1287...	3.6691...
FInancial	0	0.1314414...	0.2079738...	0.0979738...	0.0979...	0.0979738...	0.2079...	0

initialisation

FeedBack Values

Previous | Next

FInancial

New Max Value: 0.0980

New Min Value: 0.0400

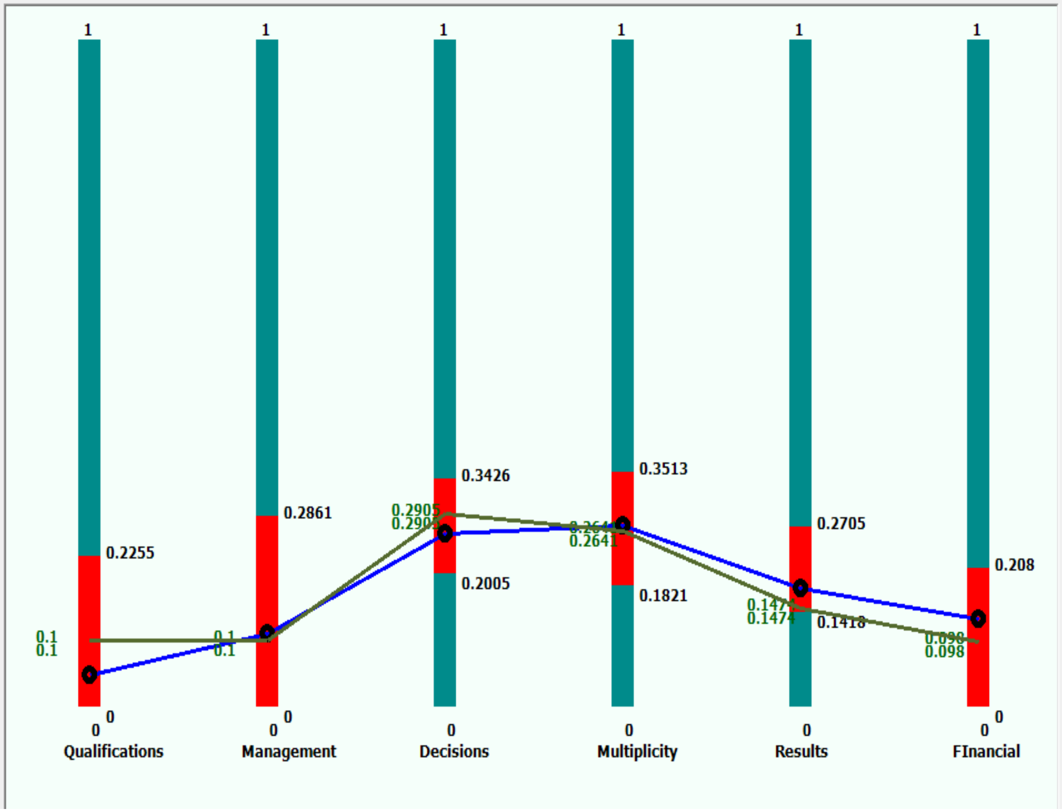
Update Values

Changes

Criterion	New Min	New M...
Qualific...	0.1	0.2254...
Manage...	0.1	0.1961...
Decisio...	0.2904...	0.3425...
Multipli...	0.2721...	0.3513...
Results	0.1418...	0.1805...
FInancial	0.04	0.0979...

Accept and Solve

Save | Cancel



Robustness Indices

S Init:	0.1933
Sd Init	0.2006
S new:	0
Sd new:	0
Snew/Sinit(%):	0.001
(1-Snew/Sinit)(%):	99.999
ASI Index:	1
ASI Initial	0.904

(•) Parallel Graph

( ) Star Graph

Pairwise prioritisation of criteria

Criteria	Min()	BC	MaxV	New Min	New BC	New Max	µi	New µi
Qualifications	0	0.0476520...	0.2254857...	0	0.0388...	0.1055781...	0.2254...	0.1055...
Management	0	0.1095381...	0.2861218...	0.2312059...	0.2535...	0.2835755...	0.2861...	0.0523...
Decisions	0.2004964...	0.2603827...	0.3425688...	0.2212059...	0.2320...	0.2509553...	0.1420...	0.0297...
Multiplicity	0.1821378...	0.2724103...	0.3513414...	0.1821371...	0.1936...	0.2105824...	0.1692...	0.0284...
Results	0.1418190...	0.1785752...	0.2705225...	0.1945055...	0.2059...	0.2269896...	0.1287...	0.0324...
FInancial	0	0.1314414...	0.2079738...	0.0395081...	0.0758...	0.1194547...	0.2079...	0.0799...

Initial

Set Priorities

Previous Next

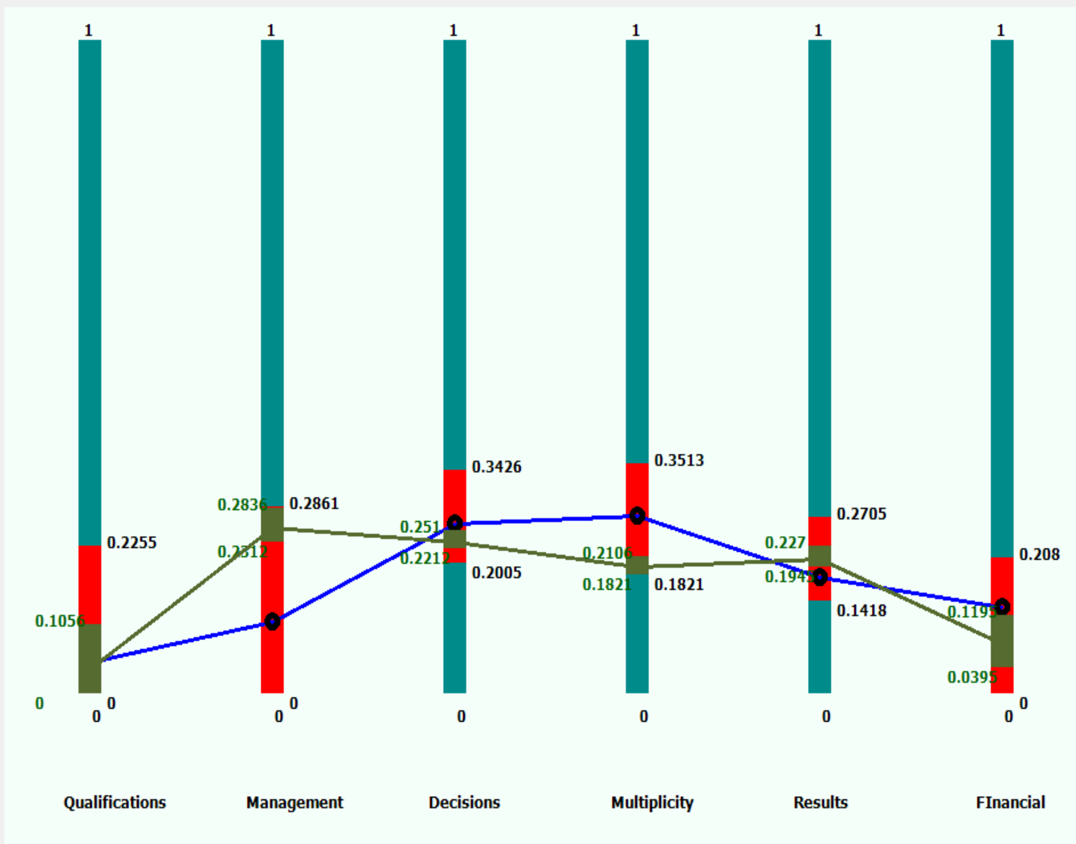
Multiplicity ○ Preference >> ◉ Preference << ○ Neutral Results

Threshold 0.01

Accept and Solve

Save

Cancel



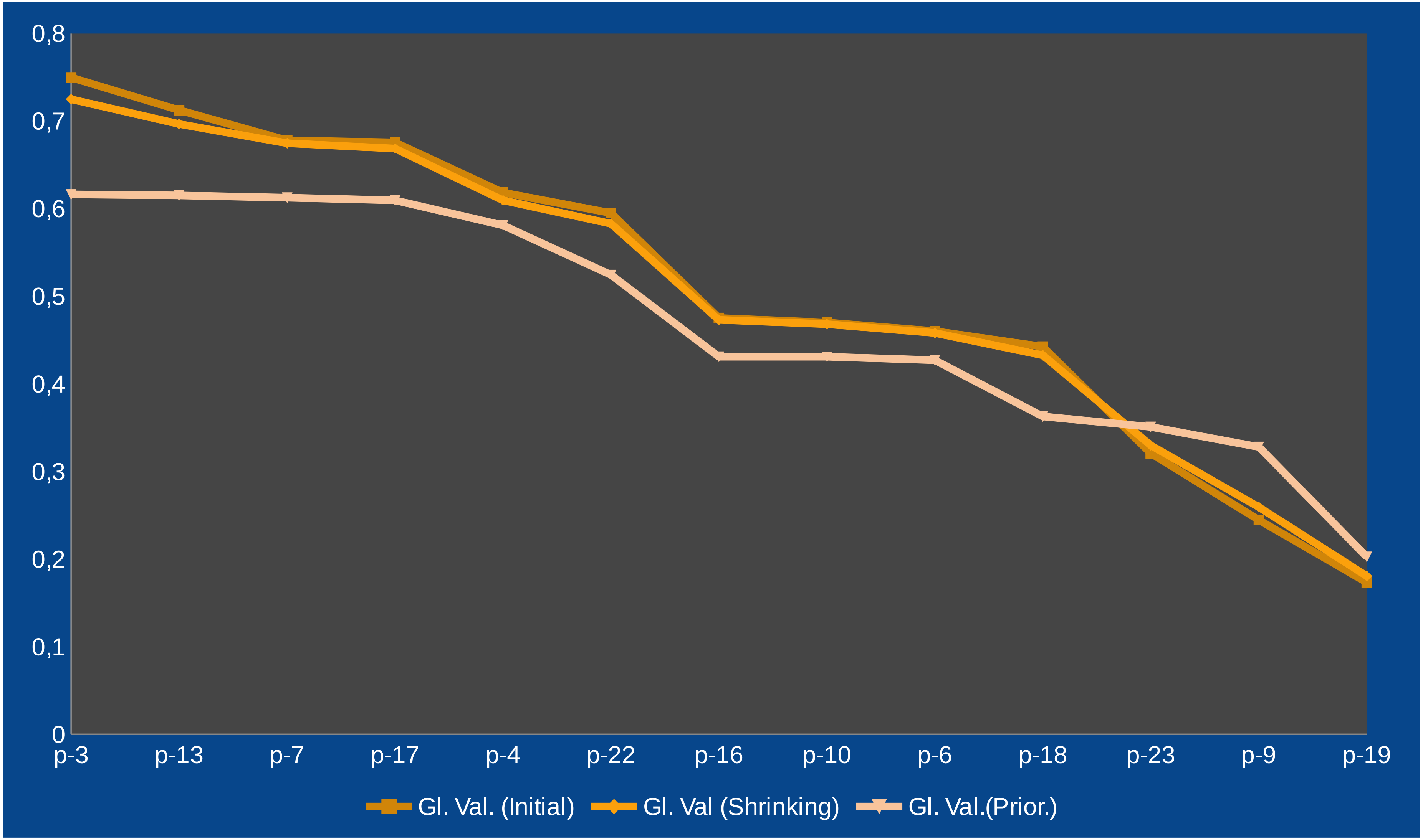
No	Crit 1	Crit 2	Pref/nce
1	Qualificati...	Management	
2	Qualificati...	Decisions	
3	Qualificati...	Multiplicity	
4	Qualificati...	Results	
5	Qualificati...	FInancial	
6	Manage...	Decisions	P
7	Manage...	Multiplicity	
8	Manage...	Results	
9	Manage...	FInancial	
10	Decisions	Multiplicity	
11	Decisions	Results	P
12	Decisions	FInancial	N.A.
13	Multiplicity	Results	~P

S init	0.1933
Sd init:	0.2006
S new:	0.0548
Sd New:	0.062
Snew/Sinit (%):	28.336
(1-Snew/Sinit(%):	71.664
ASI Index:	0.978
ASI init:	0.904

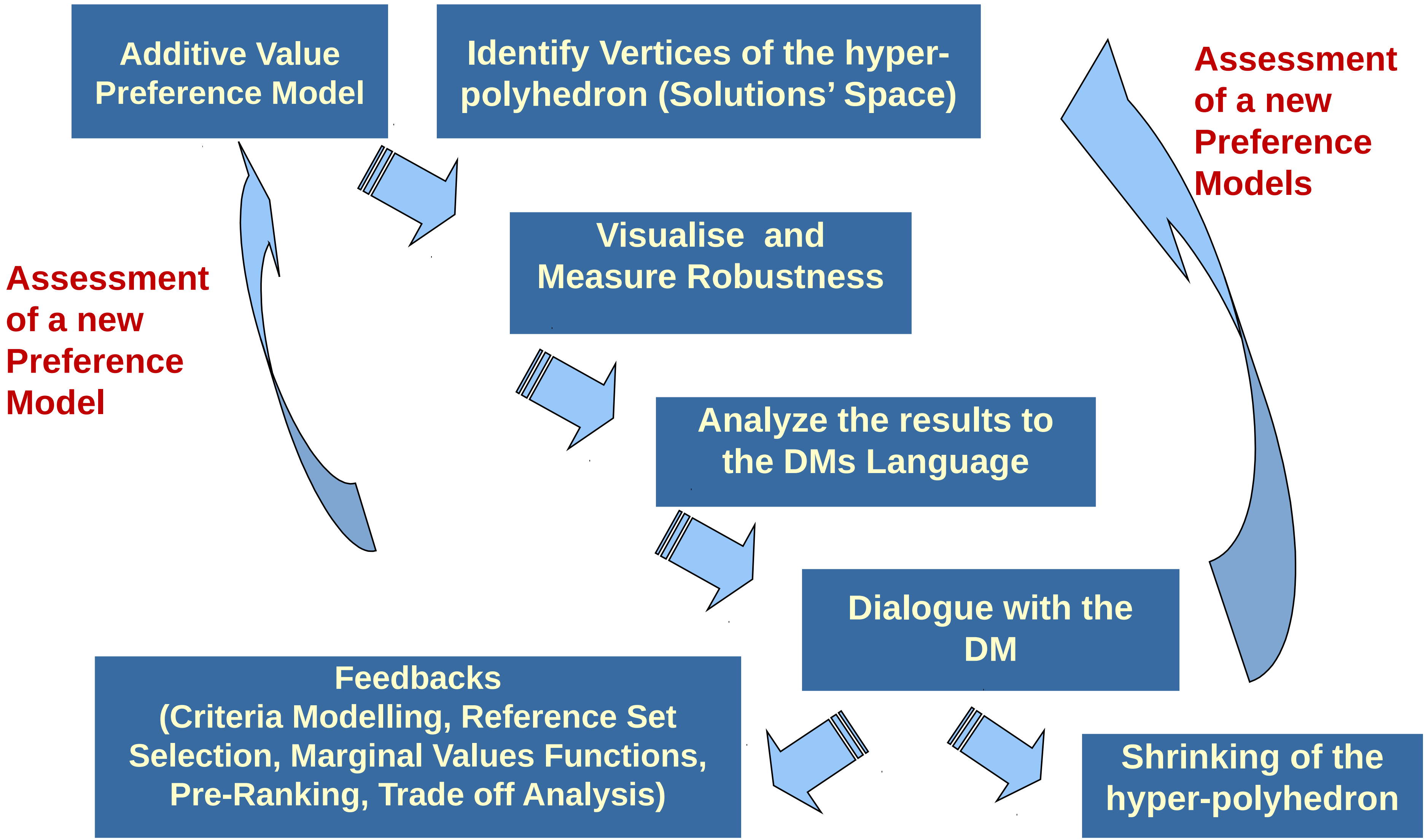
○ Parallel Diagr

○ Star Diagr.

Comparison of initial and revised global values



Steps of the proposed approach (Feedbacks)



Conclusions - Perspectives

- Preference Models with low robustness include useful information about the preferences' structure of DMs.

- There is a need to explore the nature of the low robustness and exploit it.

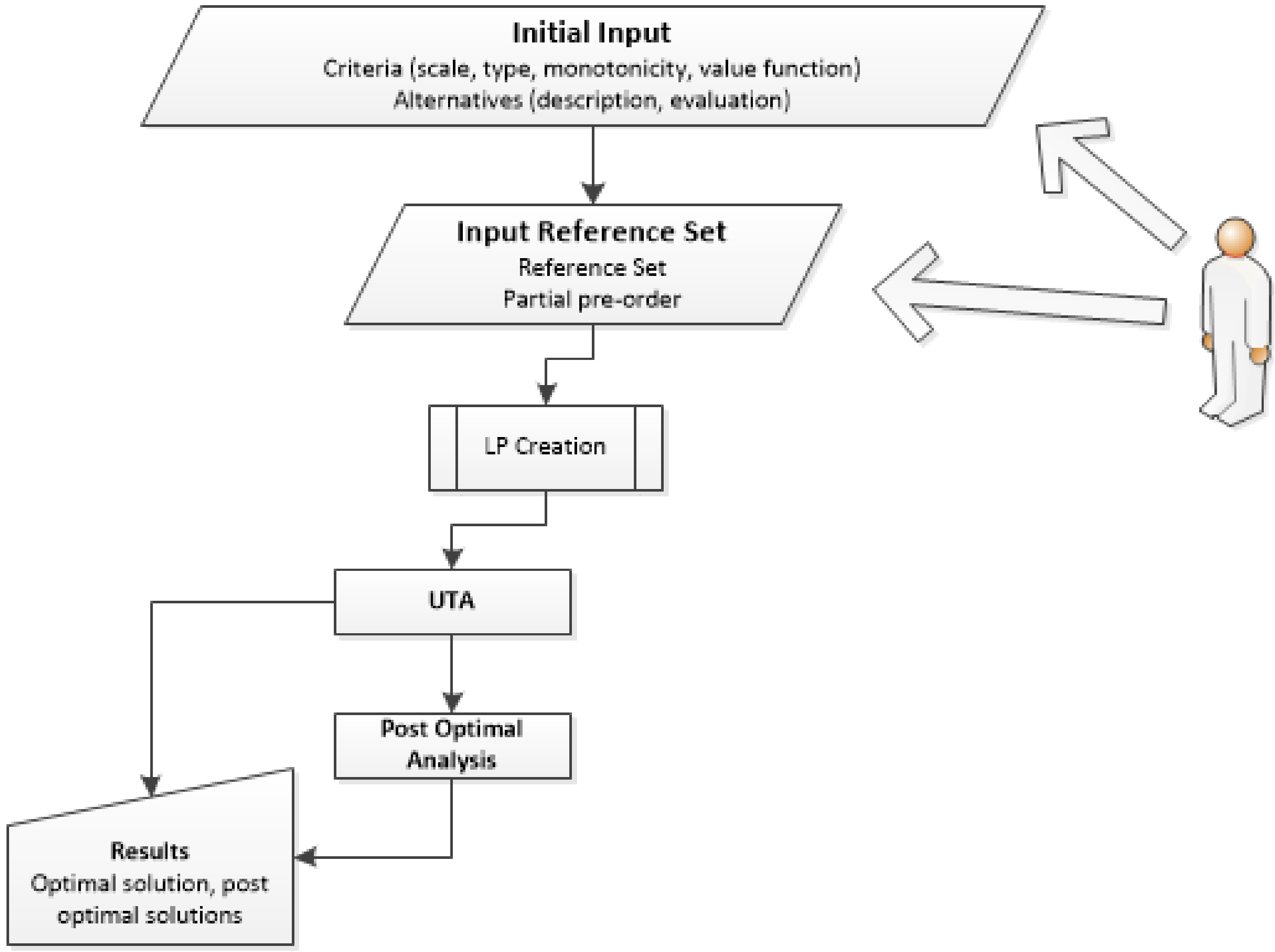
- Visualisation and robustness measures provide a better knowledge of the preference models and can support the analysis of the DM' preference structures.

- The new proposed interactive feedbacks could enrich the existing tools of D-A approach for detecting representative preference model with a better robustness.

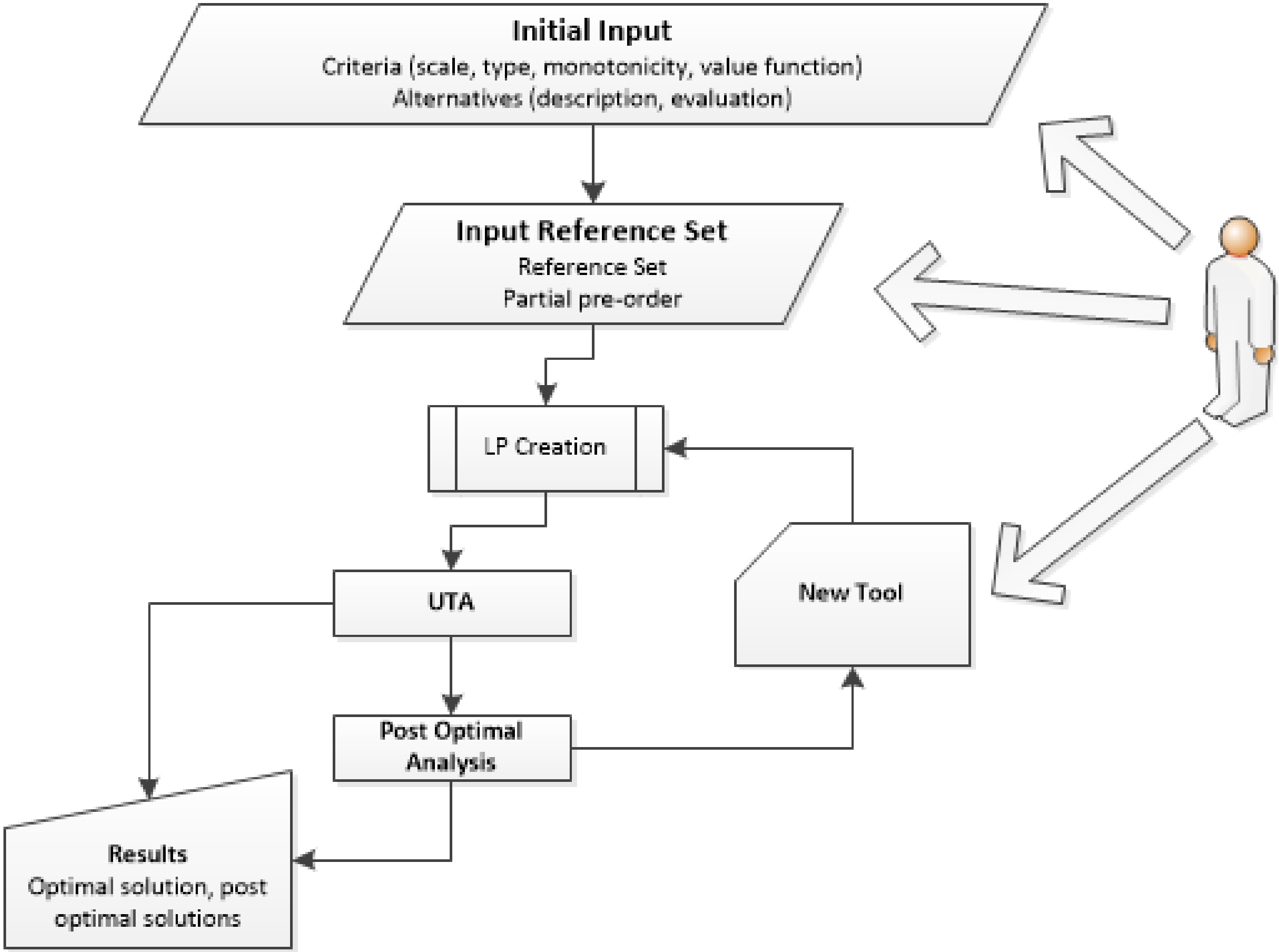
Towards a new system

- Supports the visualisation of n-dimensional spaces in 3d and 2d form
- Incorporated in MINORA and MIIDAS systems
- Supports interactive feedbacks for the scrutiny of the hyper-polyhedra
- Aims:
  - To be included as basic component (module) in MINORA and MIIDAS systems
  - Simple and easy way to present the robustness of the assessed preference structures
  - Acquire knowledge about preference models' structures and support the decision making process
  - Lead to more robust preference models through intervention on the preference models utilising addition preference information.

Using MINORA DSS



Using MINORA DSS with new tool



Thank you!

Questions