Sparse + smooth decomposition models for multi-temporal SAR images

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3 - CNES

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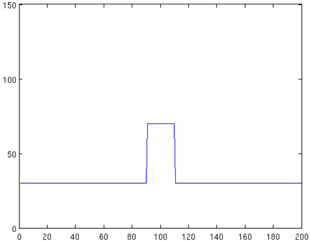
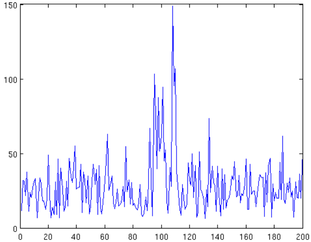
Florence Tupin[1] Loïc Denis[2]

Context

- Speckled images (Rayleigh multiplicative model when considering amplitude images):



Input signal



Speckle ($L = 1$)

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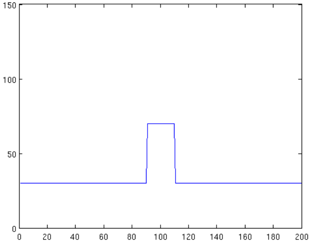

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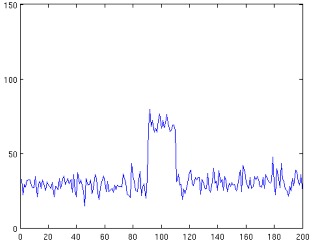

Additive Gaussian noise ($\sigma = 5$)

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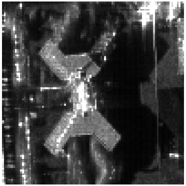
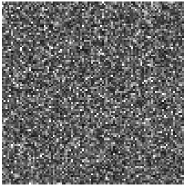


Original scene

X



Rayleigh noise (L=3)

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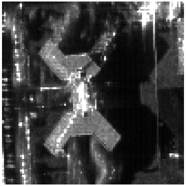

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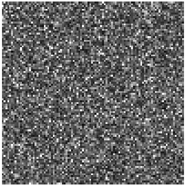

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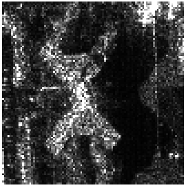

Noisy scene

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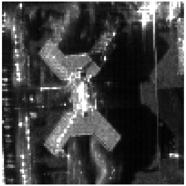

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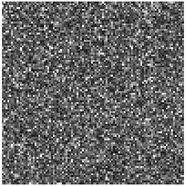

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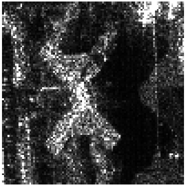

Noisy scene

Goals

- Find regularization models.
- Use multi-temporal information.

Plan

Model presentation

- Scene to estimate $\mathbf{U} = \{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$.
- Observed images $\mathbf{V} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.
- In a Markov Random Field framework:

$$\begin{aligned}
\hat{\mathbf{U}} &= \arg\min_{\mathbf{U}} -\log\left(P(\mathbf{U}|\mathbf{V})\right) \\
&= \arg\min_{\mathbf{U}} -\log\left(P(\mathbf{V}|\mathbf{U})\right) - \log\left(P(\mathbf{U})\right) \\
&= \arg\min_{\mathbf{U}} \mathcal{E}(\mathbf{U})
\end{aligned}$$

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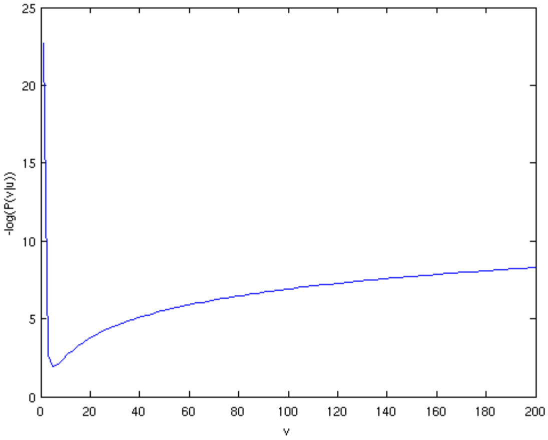
\end{aligned}
$$

Likelihood-term definition

- Likelihood distribution can be considered separable:

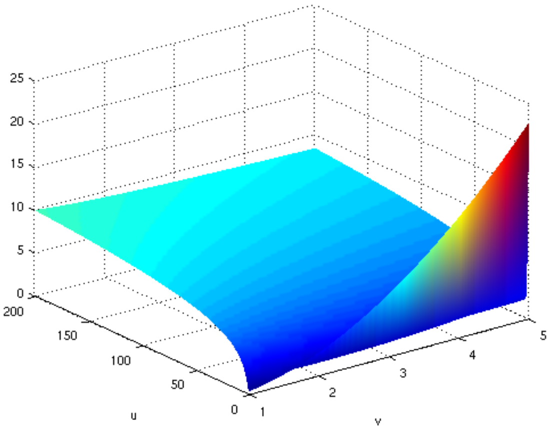
$$
\begin{aligned}
-\log\left(P(\mathbf{V}|\mathbf{U})\right) &= -\log\left(\prod_{t=1}^{n}\prod_{i\in\Omega}\mathrm{p}(\mathbf{v}_t(i)|\mathbf{u}_t(i))\right)\\
&= -\log\left(\prod_{t=1}^{n}\prod_{i\in\Omega}\frac{2\,\mathbf{v}_t(i)}{\mathbf{u}_t^2(i)}\exp\left(\frac{-\mathbf{v}_t^2(i)}{\mathbf{u}_t^2(i)}\right)\right)\\
&= \sum_{t=1}^{n}\sum_{i\in\Omega}\left(-\log(2\mathbf{v}_t(i)) + 2\log(\mathbf{u}_t(i)) + \frac{\mathbf{v}_t^2(i)}{\mathbf{u}_t^2(i)}\right)\\
&= \mathrm{DT}(\mathbf{V},\mathbf{U})
\end{aligned}
$$

Likelihood-term definition



Conditional negative log likelihood of Rayleigh distribution for $u = 5$

Likelihood-term definition



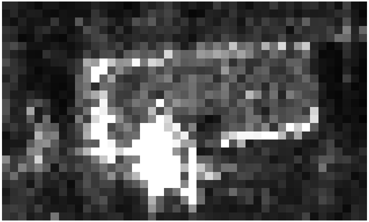
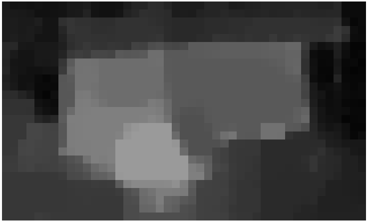
Conditional negative log likelihood of Rayleigh distribution

Prior definition

- A widely used regularization for noise reduction is the anisotropic total variation (TV)

$$-\log \mathrm{p}(\mathbf{U}) = \mathrm{TV}_{3D}(\mathbf{U})$$
$$= \sum_{t,(i,j)\in\mathcal{C}} |\mathbf{u}_t(i) - \mathbf{u}_t(j)| + \sum_{t,i\in\Omega} |\mathbf{u}_t(i) - \mathbf{u}_{t+1}(i)|$$

- But it is not adapted to scatterers present in SAR images ([Denis et al., 2009]):

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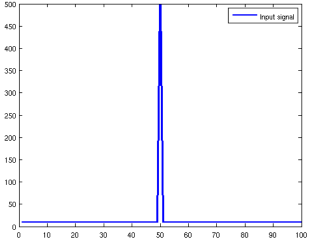

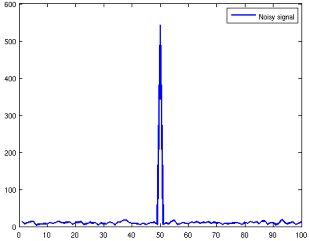

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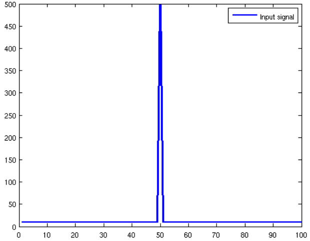

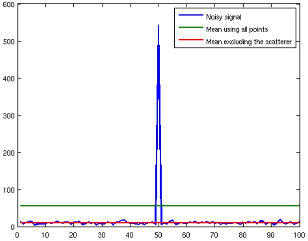

Proposed model: sparse + smooth

- The scene is modeled as the sum of 2 components:

$$\mathbf{U} = \{\dots, \mathbf{u_{BV}}_t + \mathbf{u_S}_t, \dots\} = \mathbf{U_{BV}} + \mathbf{U_S}$$

- $\mathbf{u_{BV}}_t$ is a component with bounded variations, representing the background;
- $\mathbf{u_S}_t$ is a sparse component, representing the bright scatterers.

Proposed model: sparse + smooth

- Rewrite the prior:

$$-\log(\mathrm{p}(\mathbf{U})) = -\beta_{BV} \log \mathrm{p}(\mathbf{U_{BV}}) - \beta_S \log \mathrm{p}(\mathbf{U_S})$$

- We want a low total variation on the background:

$$-\log \mathrm{p}(\mathbf{U_{BV}}) = \mathrm{TV}_{3\mathrm{D}}(\mathbf{U_{BV}})$$

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- We want a low total variation on the background:

$$-\log \mathrm{p}(\mathbf{U_{BV}}) = \mathrm{TV}_{3\mathrm{D}}(\mathbf{U_{BV}})$$

- Sparsity of the scatterers is achieved using the $\mathrm{L0}$ pseudo-norm:

$$-\log \mathrm{p}(\mathbf{U_S}) = \|\mathbf{U_S}\|_0 \,.$$

Model: final form

- We have to optimize:

$$\hat{\mathbf{U}} = \arg\min_{\mathbf{U}} \mathcal{E}(\mathbf{U})$$
$$= \arg\min_{\mathbf{U}} -\log\left(P(\mathbf{V}|\mathbf{U})\right) - \log\left(P(\mathbf{U})\right)$$

Model: final form

- We have to optimize:

$$\begin{aligned}\hat{\mathbf{U}} &= \arg\min_{\mathbf{U}} \mathcal{E}(\mathbf{U}) \\ &= \arg\min_{\mathbf{U}} -\log\left(P(\mathbf{V}|\mathbf{U})\right) - \log\left(P(\mathbf{U})\right) \\ &= \sum_{t=1}^{n} \left(\mathrm{DT}(\mathbf{v}_t, \mathbf{u_{BV}}_t, \mathbf{u_S}_t) + \beta_S \mathrm{L0}(\mathbf{u_S}_t)\right) + \beta_{BV} \mathrm{TV}_{\mathrm{3D}}(\mathbf{U_{BV}})\end{aligned}$$

- Non convex problem because of the data term and the pseudo norm L0.

Solving the sub-problem for a fixed $\mathbf{U_{BV}}$

- We need to solve the problem for a fixed $\mathbf{U_{BV}}$:

$$\widehat{\mathbf{U}_S}(\mathbf{U_{BV}}) = \arg\min_{\mathbf{U}_S} \sum_{t=1}^{n} \left(\mathrm{DT}(\mathbf{v}_t, \mathbf{u_{BV}}_t, \mathbf{u_S}_t) + \beta_S \mathrm{L0}(\mathbf{u_S}_t) \right) + \beta_{BV} \mathrm{TV}_{3D}(\mathbf{U_{BV}})$$

- Since terms are separable, we can solve for each pixel at each date:

$$\widehat{\mathbf{u_S}_t(i)}(\mathbf{u_{BV}}_t(i)) = \begin{cases} \mathbf{u_S}_t(i)^\star & \text{if } \mathrm{DT}(\mathbf{v}_t(i), \mathbf{u_{BV}}_t(i), \mathbf{u_S}_t(i)^\star) \\ & +\beta_S < \mathrm{DT}(\mathbf{v}_t(i), \mathbf{u_{BV}}_t(i), 0) \\ 0 & \text{otherwise} \end{cases}$$

With $\mathbf{u_S}_t(i)^\star = \arg\min_{\mathbf{u_S}_t(i)} \mathrm{DT}(\mathbf{v}_t(i), \mathbf{u_{BV}}_t(i), \mathbf{u_S}_t(i))$ which can be found analytically.

Solving the whole problem

- Problem only depending $\mathbf{U_{BV}}$:

$$\arg\min_{\mathbf{u_{BV}}} \mathcal{E}(\mathbf{U}) = \arg\min_{\mathbf{u_{BV}}} \sum_{t=1}^{n} \mathrm{DT}(\mathbf{v}_t, \mathbf{u_{BV}}_t, \widehat{\mathbf{u_S}_t}(\mathbf{u_{BV}}_t)) + \beta_S \|\widehat{\mathbf{u_S}_t}(\mathbf{u_{BV}}_t)\|_0 + \beta_{BV} \mathrm{TV}_{3D}(\mathbf{U_{BV}})$$

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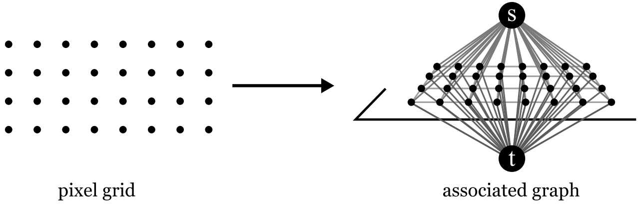
- The first two terms are separable and the third one is convex and involves only pairs of pixel values $\Longrightarrow$ [Ishikawa, 2003]

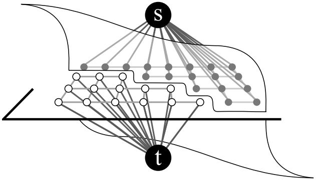
Graph-cut optimization

The pixel grid is mapped to a graph with two terminal nodes:



Graph-cut optimization

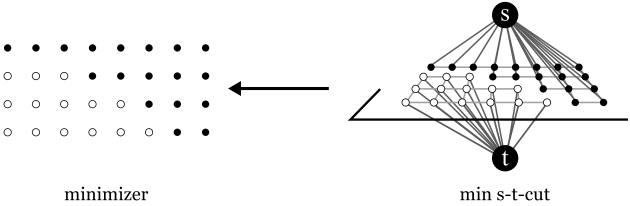
A minimum s-t-cut is computed:



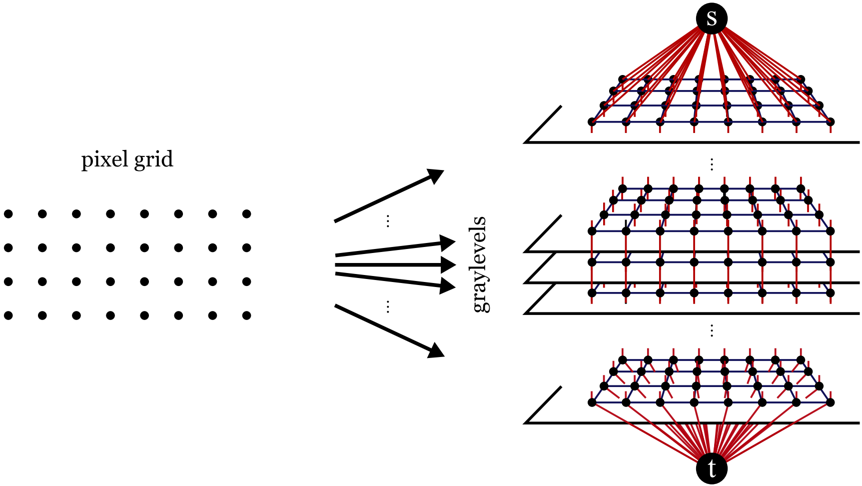
min s-t-cut

Graph-cut optimization

The cut is interpreted as a solution of the original problem:

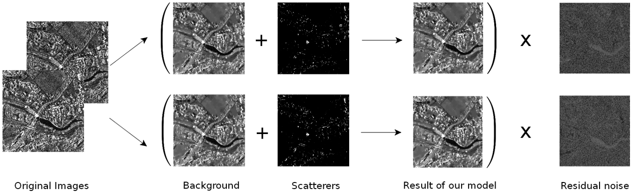


Graph-cut optimization



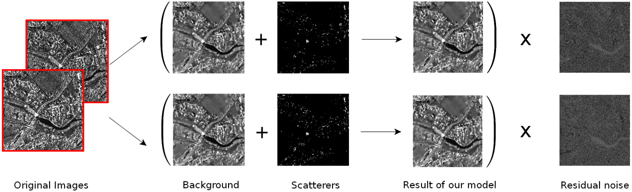
Ishikawa's graph for multi-valued images
[Ishikawa PAMI2003]

Results



Decomposition of an image of Saint-Gervais acquired by TerraSAR-X. Thanks to the German Aerospace Agency (DLR) for the images (project MTH0232 and LAN1746).

Results



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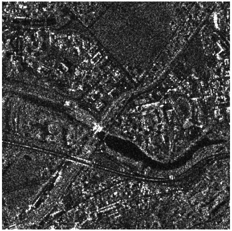

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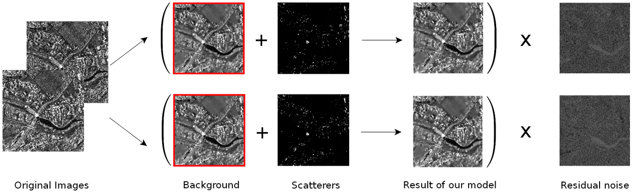

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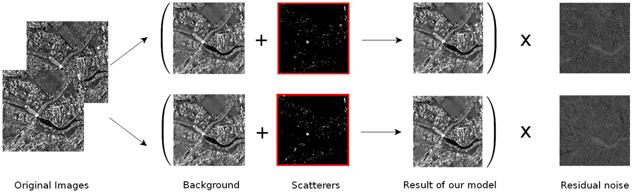

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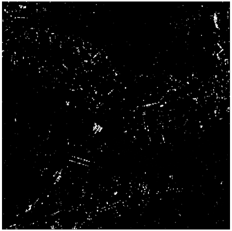

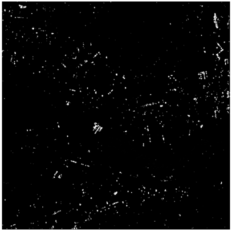

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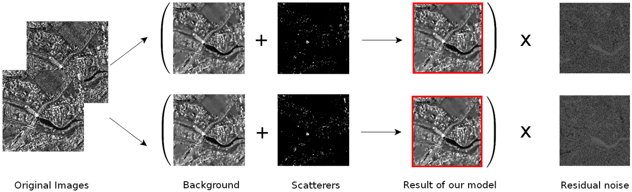

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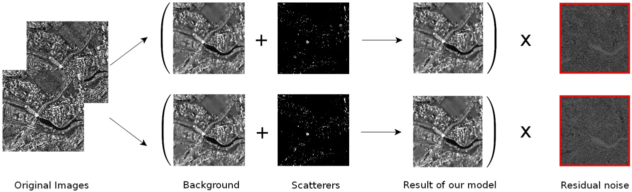

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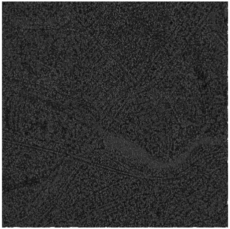

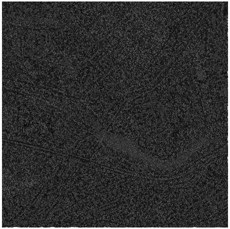

Plan

Method

- From a series of observed images $\mathbf{V}$, we obtain decomposition $\{(\mathbf{u_{BV}}_1, \mathbf{u_S}_1), \ldots, (\mathbf{u_{BV}}_n, \mathbf{u_S}_n)\}$
- Keep the binarized version of the scatterers $\{\mathbf{u_S}_t^{bin}\}$.

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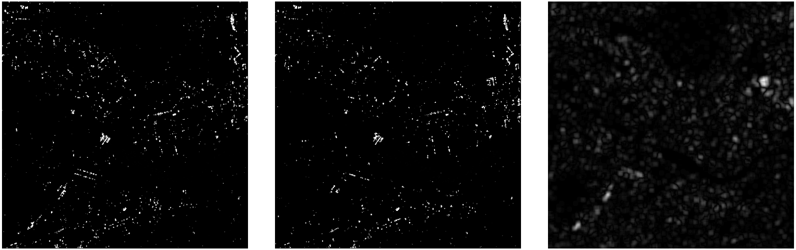
- To cope with non-stability of the scatterers, construct:

$$T(i) = \left| \sum_{\delta} \mathbf{u_S}_1^{bin}(i+\delta) - \sum_{\delta} \mathbf{u_S}_2^{bin}(i+\delta) \right|$$



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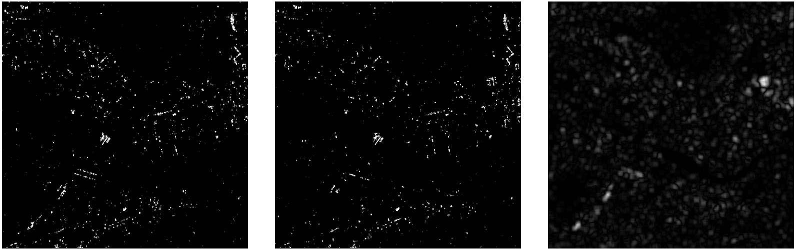
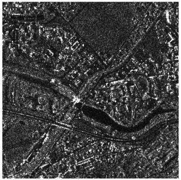


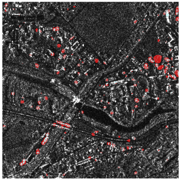
- $T(i)$ is then thresholded to obtain the changes.

Results

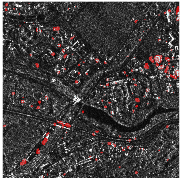


Input image 1.

Input image 26.



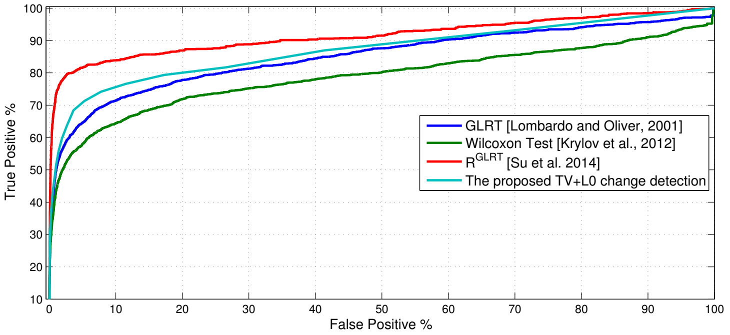
Changes on image 1.



Changes on image 26.

Change detection results using the proposed method on images of Saint-Gervais series. Regions with changes that have been detected are indicated in red.

Comparison with other algorithms



False positive alarm versus true positive curves of various change detection algorithms ([Su et al., 2014], [Lombardo and Oliver, 2001], [Krylov et al., 2012])

Plan

Conclusion

- We introduced a decomposition model that:
  - Combines TV regularization with an L0 pseudo-norm suitable to SAR images.
  - Is able to take advantage of multi-temporal SAR series.
  - Can be used in a change detection application.
- Future work:
  - A change detection algorithm using all components.
  - Using optimization methods requiring less memory (see [Shabou et al., 2009]).

Questions ?

Bibliographie

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