Accounting for missing data in image data assimilation

Vincent Chabot, Maëlle Nodet and Arthur Vidard

Univ. Grenoble Alpes / INRIA / Météo France

Multitemp 2015

Outline

Introduction

Observation error covariance modelling

Dealing with missing data

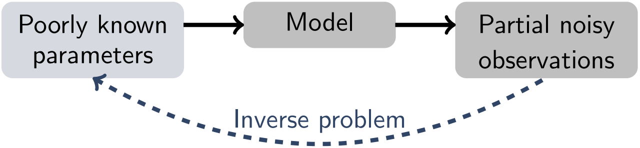
What is data assimilation?

Direct problem:

What is data assimilation?

Direct problem:

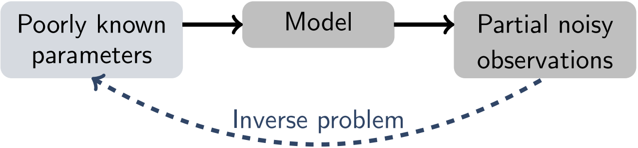
Inverse problem (called Data Assimilation in the NWP context):



What is data assimilation?

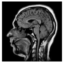
Direct problem:

Inverse problem (called Data Assimilation in the NWP context):



Famous examples:

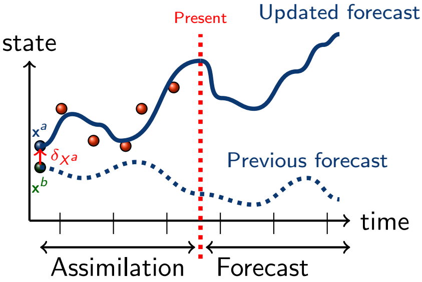

Sherlock Holmes (observable consequences and clues → what *really* happened)


Medical imaging (external measurements → tissue properties)


Numerical Weather Prediction (NWP) (observations → initial condition to produce a weather forecast)

Variational data assimilation

Numerical Weather Prediction (NWP) (observations $\rightarrow$ initial condition to produce a weather forecast)



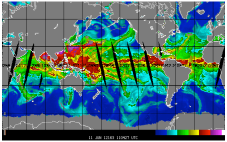
Optimal initial condition: x_0^a minimizer of the cost function:

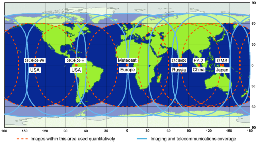
$$J(x_0) = \underbrace{\cdots}_{\text{regularisation}} + \frac{1}{2}\sum_i \left\| H(\mathcal{M}_{t_0 \rightarrow t_i}(x_0)) - y_i^o \right\|_{\mathbf{R}}^2$$

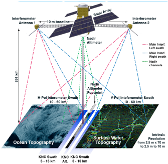
regularisation misfit to observations

$\left\|x\right\|_{\mathbf{R}}^2 = x^T \mathbf{R}^{-1} x$ $\mathbf{R}$: observations error covariance matrix

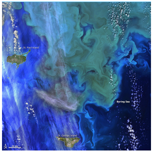
R to account for heterogeneous information







- Heterogeneous in quality, quantity, nature.
- Sparse under the surface
- Continuously increasing number.



Observation error covariances modelling

$$J(x_0) = \dots + \frac{1}{2}\sum_i \left\| H(\mathcal{M}_{t_0\to t_i}(x_0)) - y_i^o \right\|_{\mathbf{R}}^2$$

$$\text{with } \|x\|_{\mathbf{R}}^2 = x^T \mathbf{R}^{-1} x$$

Dense-field observations are large, therefore **R** is big.

- computing issue: var. assim. requires its inverse
- storage issue: even if sparse, its inverse is not
- $\rightarrow$ diagonal **R** !

R for dense-field observations:

- errors correlated in space
- in general, diagonal approx + obs. thinning
- thinning: small scale information is lost!

Idea of this work: use the sparseness of multiscale decomposition to try and **keep the diagonal approximation** for **R** alive (still **accounting for some spatial correlation**)

Outline

Introduction

Observation error covariance modelling

Dealing with missing data

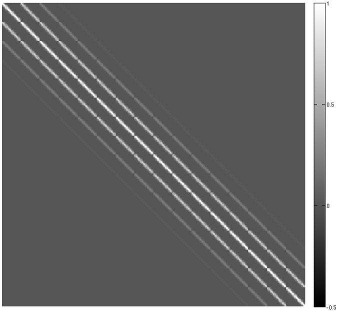
UNIVERSITÉ GRENOBLE ALPES

True correlation matrices

Assuming Gaussian statistics and correlated noise

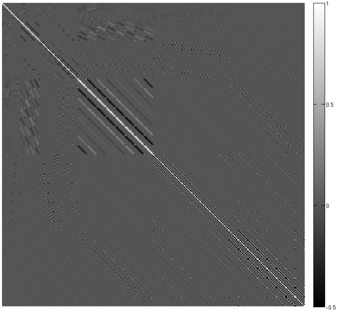
$$\mathbf{C}_{wav} = W \mathbf{C}_{pix} W^T$$

True $\mathbf{C}_{pix}$ (not diagonal)



Pixels

True $\mathbf{C}_{wav}$ ("more" diagonal)



Daubechies

$\Rightarrow$ Diagonal approximation seems "less wrong" in wavelet space...

Change of variable into wavelet space

Trick to keep **R** diagonal

Write the cost function as

$$(\mathbf{y}-\mathcal{H}(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y}-\mathcal{H}(\mathbf{x})) = (\mathbf{y}-\mathcal{H}(\mathbf{x}))^T\mathbf{A}^T\mathbf{D}_{\mathbf{A}}^{-1}\mathbf{A}(\mathbf{y}-\mathcal{H}(\mathbf{x}))$$

3- Change variable back to pixel space ($\mathbf{A}^T$)

2- Diagonal cov. mat. in wav. spcace ($\mathbf{D}_{\mathbf{A}}^{-1}$)

1- Change variable to wavelet space ($\mathbf{A}$)

UNIVERSITÉ GRENOBLE ALPES

Change of variable into wavelet space

Trick to keep R diagonal

Write the cost function as

$$(\mathbf{y}-\mathcal{H}(\mathbf{x}))^T\mathbf{R}^{-1}(\mathbf{y}-\mathcal{H}(\mathbf{x})) = (\mathbf{y}-\mathcal{H}(\mathbf{x}))^T\mathbf{A}^T\mathbf{D}_{\mathbf{A}}^{-1}\mathbf{A}(\mathbf{y}-\mathcal{H}(\mathbf{x}))$$

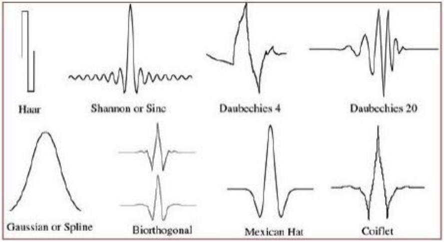
3- Change variable back to pixel space → $\mathbf{A}^T$

2- Diagonal cov. mat. in wav. spcace → $\mathbf{D}_{\mathbf{A}}^{-1}$

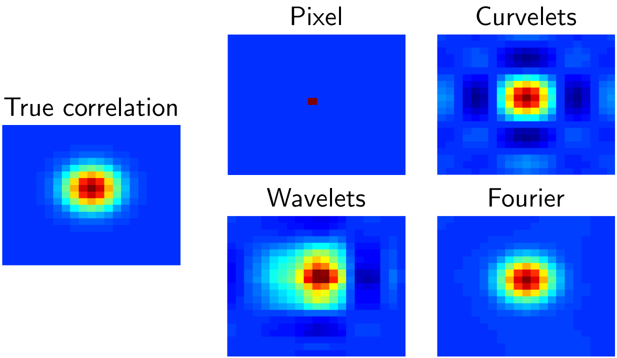
1- Change variable to wavelet space → $\mathbf{A}$

Why does it work?

Wavelet basis contains spatial correlation:



Correlations obtained when using diagonal covariance matrices



UNIVERSITÉ GRENOBLE ALPES

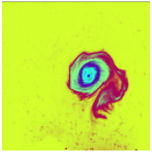
Impact of error statistics

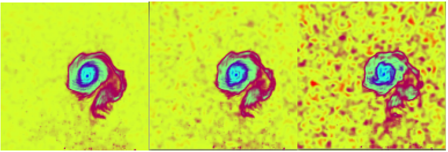
Numerical results (1)

Mean over 10 experiments of the residual error after image assimilation:

Signal to Noise Ratio (SNR)	26.8 dB (small noise)	20.8 dB	14.8 dB (high noise)
Pixel – Scalar	15.2%	21.8%	36.8%
Curvelets – Diag	8.1%	7.7%	8.3%
Wavelets D_8 – Diag	7.0%	7.5%	9.1%
Fourier – Diag	7.3%	7.3%	7.3%

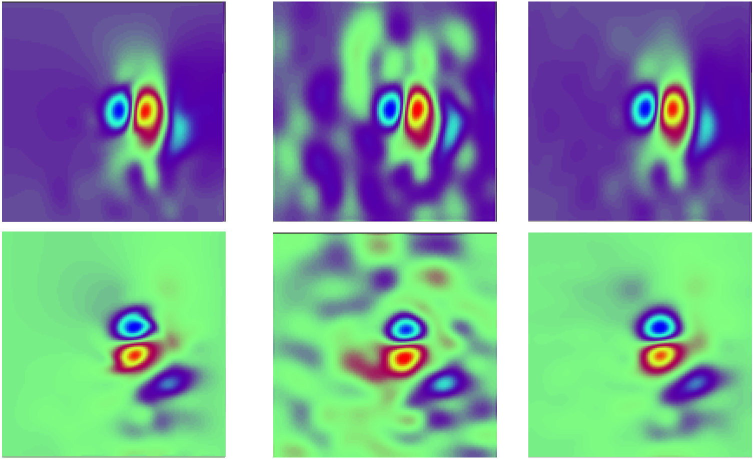
Perfect data // Noisy images, SNR 26.8 dB, 20.8 dB and 14.8 dB:





Impact of error statistics

Numerical results (2)



True velocities   Diag – pixel   Diag – wavelets

Outline

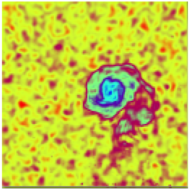
Introduction

Observation error covariance modelling

Dealing with missing data

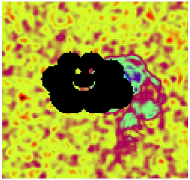
Managing missing data

Many images may suffer from missing data, as for example ocean colour masked by a passing cloud.

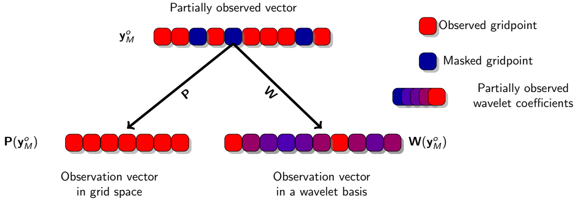


Managing missing data

Many images may suffer from missing data, as for example ocean colour masked by a passing cloud.



Managing occultations



Grid/pixel space: easy!

$$\|H(x) - y^{seen}\|^2_{R^{seen}_{grid}}$$

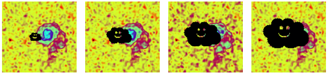
where H includes a projection (masking).

Wavelet space: tricky!

- naive approach: mask out the synthetic observation and take it as normal image.
- not so naive approach: account for the information content of each wav. coefficients

Managing occultations (2)

the more holes, the less Emmental cheese. . .



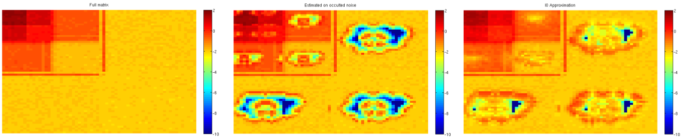
Conflicting issues:

- missing data: no error

  $\Longrightarrow$ correction: deflate error statistics

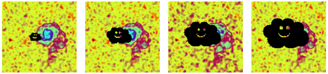
- missing data: more discontinuity in the signal + perturbed small scale coefficients

  $\Longrightarrow$ correction: inflate error statistics



Managing occultations (2)

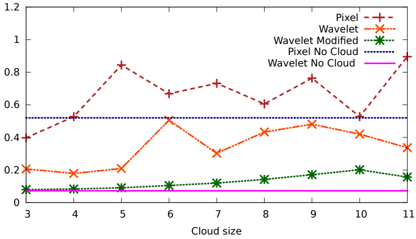
the more holes, the less Emmental cheese...



Conflicting issues:

- missing data: no error
  $\Longrightarrow$ correction: deflate error statistics
- missing data: more discontinuity in the signal + perturbed small scale coefficients
  $\Longrightarrow$ correction: inflate error statistics

normalized RMS error in meridional velocity



Thank you for your attention

References:

- V. Chabot, M. Nodet, N. Papadakis and A. Vidard, *Assimilation of images in the presence of observation error*, Tellus A, 2015
- V. Chabot, *Etude de représentations parcimonieuses des statistiques d'erreur d'observation pour différentes métriques. Application à l'assimilation d'images*, PhD thesis manuscript, Université de Grenoble, 2014
- G. Desroziers, L. Berre, B. Chapnik, and P. Poli. *Diagnosis of observation, background and analysis-error statistics in observation space*, Quarterly Journal of the Royal Meteorological Society, 2005.
- L. M. Stewart, S. Dance, and N. K. Nichols. *Data assimilation with correlated observation errors: experiments with a 1-d shallow water model*, Tellus A, 2013.
- O. Titaud, A. Vidard, I. Souopgui, and F.-X. Le Dimet. *Assimilation of image sequences in numerical models*, Tellus A, 2010.

UNIVERSITÉ GRENOBLE ALPES

Outline

Correlations

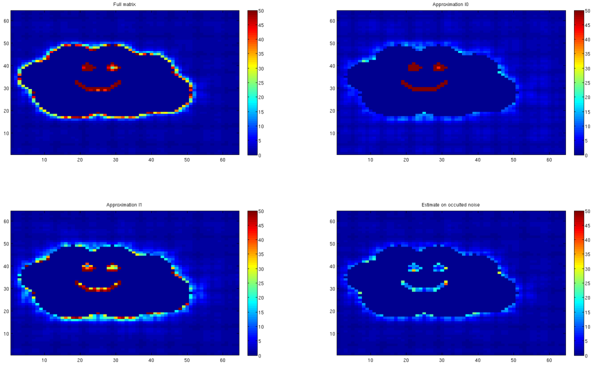
Numerical setup

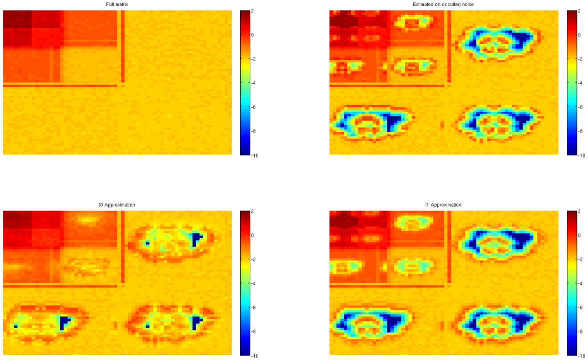
Observations thinning

Multiscale analysis and wavelets

Observation errors representation

Occultations formula


Full matrix
Approximation l0
Approximation l1
Estimate on occulted noise


Full matrix
Estimated on occulted noise
I0 Approximation
I1 Approximation
2
0
-2
-4
-6
-8
-10

Outline

Correlations

Numerical setup

Observations thinning

Multiscale analysis and wavelets

Observation errors representation

Occultations formula

Numerical setup

Optimisation:

- Background at rest ($\mathbf{x^b} = (\mathbf{0}, \mathbf{0}, \mathbf{h_{mean}})^\mathbf{T}$)
- Usual $\mathbf{B}^{1/2}$ change of variable with correlation built using Weaver and Courtier (2001) approach.
- Progressive (or quasi-static) minimisation technique [Luong et al.(1998), Pires et al. (1996)]
- Minimizer: M1QN3 [Gilbert and Lemarechal]
- Twin experiments...

Observations:

- Image resolution: 128×128 (same as model)
- Error correlation: 3 pixels

Outline

Correlations

Numerical setup

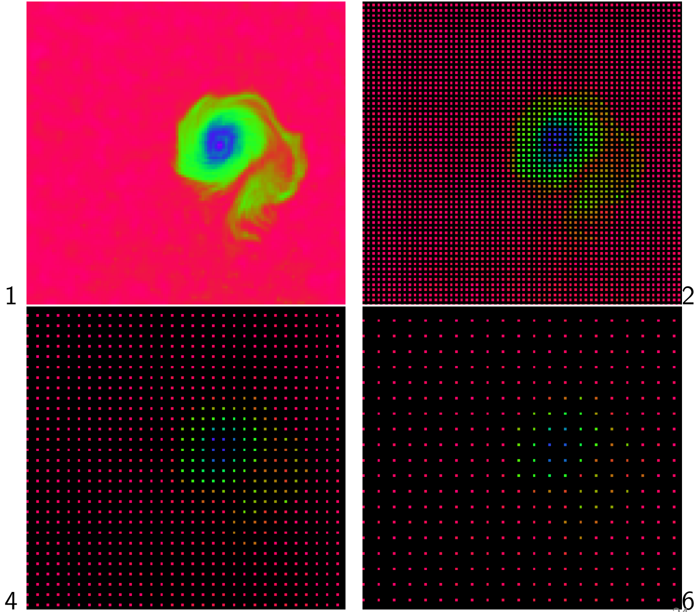
Observations thinning

Multiscale analysis and wavelets

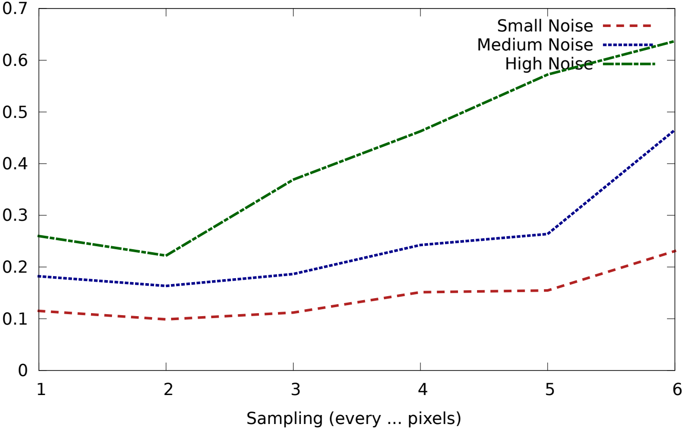
Observation errors representation

Occultations formula

What do the observations look like?



Results



(noise correlation: 3 pixels)

Outline

Correlations

Numerical setup

Observations thinning

Multiscale analysis and wavelets

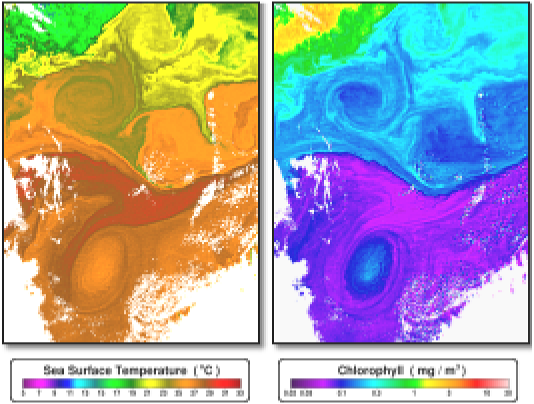
Observation errors representation

Occultations formula

Interpretation levels of images (1)

Elementary level: value at each pixel

- Large quantity of information
- Strong dependance from acquisition conditions and measuring uncertainties

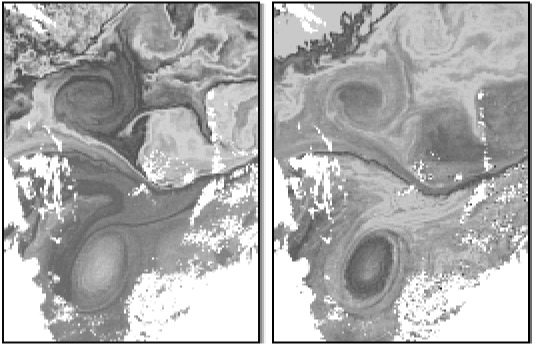


(image MODIS, source NASA)

Interpretation levels of images (2)

Structured (global) level: spacial organization of pixels

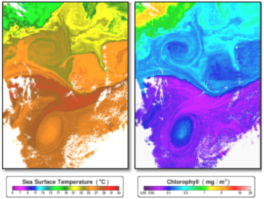
- Weak dependance from **acquisition conditions** and measuring **uncertainties**
- Dominated by the global dynamics



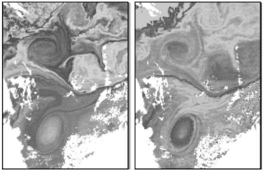
(image MODIS, source NASA)

Interpretation levels of images

Elementary level: value at each pixel



Structured (global) level: spacial organization of pixels



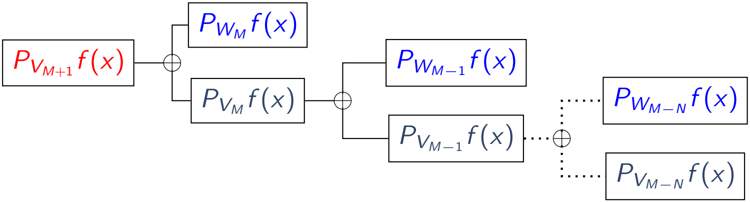
→ **Multiscale analysis to take into account structure information**

Wavelets

Mallat's Multiresolution Analysis (1)

Assume we have an image/signal at given at a fine scale $M+1$.
Let V_M be the approximation space of a given signal at scale M.
W_M (the *details* space) is defined as the orthogonal complement of V_M in V_{M+1}:

$$V_{M+1} = V_M \bigoplus W_M$$



Wavelets

Mallat's Multiresolution Analysis (1)

Assume we have an image/signal at given at a fine scale $M+1$.
Let V_M be the approximation space of a given signal at scale M.
W_M (the *details* space) is defined as the orthogonal complement of V_M in V_{M+1}:

$$V_{M+1} = V_M \bigoplus W_M$$

$$P_{V_{M+1}} f(x) = P_{W_M} f(x) \oplus P_{W_{M-1}} f(x) \oplus \begin{array}{c} P_{W_{M-N}} f(x) \\ \oplus \\ P_{V_{M-N}} f(x) \end{array}$$

V_{M+1} can be decomposed as:

$$\underbrace{P_{V_{M+1}} f(x)}_{\text{Fine signal}} = \underbrace{\sum_k c_{M-N,k} \phi_{M-N,k}(x)}_{\text{Coarse approx. in } v_{M-N}} + \sum_{r=M-N \cdots M} \underbrace{\sum_k d_{r,k} \psi_{r,k}(x)}_{\text{Details in } w_r}$$

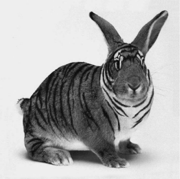
Wavelets

Mallat's Mutliresolution Analysis (2)

V_{M+1} can be decomposed as:

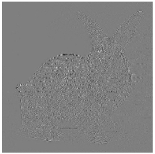
$$\underbrace{P_{V_{M+1}} f(x)}_{\text{Fine signal}} = \underbrace{\sum_k c_{M-N,k}\phi_{M-N,k}(x)}_{\text{Coarse approx. in } V_{M-N}} + \sum_{r=M-N\cdots M} \underbrace{\sum_k d_{r,k}\psi_{r,k}(x)}_{\text{Details in } W_r}$$

Original Image (scale M+1)

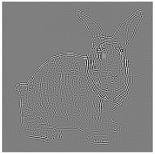


=

Details M



Details M-1

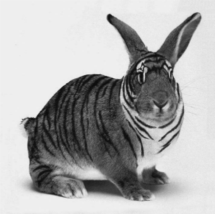


Details M-2

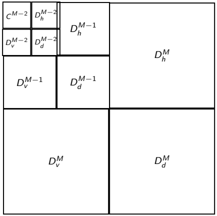
Approximation M-2

Wavelet representation

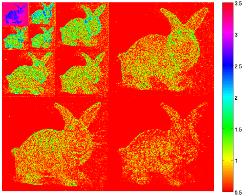
Original image



Wavelet decomposition representation



Wavelet coefficients distribution (log)



Outline

Correlations

Numerical setup

Observations thinning

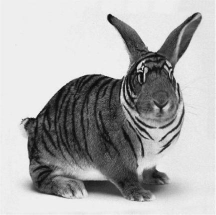
Multiscale analysis and wavelets

Observation errors representation

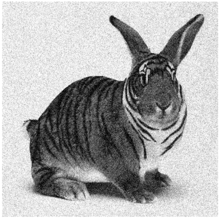
Occultations formula

Effect of noise in the wavelet space

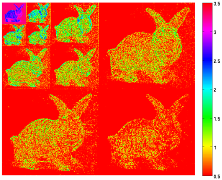
Noise free image



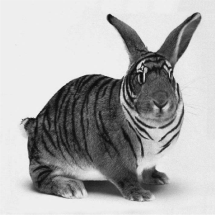
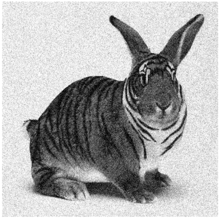
Uncorrelated noise

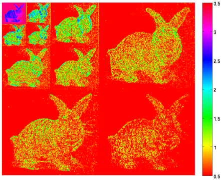


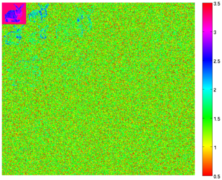
Correlated noise



Effect of noise in the wavelet space

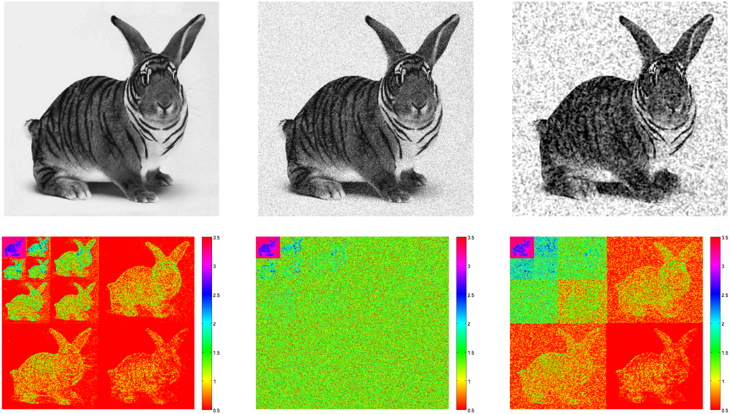
Noise free image	Uncorrelated noise	Correlated noise
		





Effect of noise in the wavelet space

Noise free image | Uncorrelated noise | Correlated noise



Outline

Correlations

Numerical setup

Observations thinning

Multiscale analysis and wavelets

Observation errors representation

Occultations formula

wavelet decomposition:

$$c^{j-1}[n] = \sum_p h[p-2n]c^j[p]$$

$$d^{j-1}[n] = \sum_p g[p-2n]d^j[p]$$

some attempt to account for missing data:

$$\tilde{\sigma}^2_{d_i} = (\sigma^2_{d_i} + \alpha_{d_i}\sigma^2_{mean}) \times I$$

where

$$\alpha_{d_i} = \left| \frac{\sum g^{occ}[p-2n]}{\sum |g^{occ}[p-2n]|} \right|$$

I^0 number of observed grid point
I^1 weighted number of observed grid point