A gap-filling method to reconstruct incomplete SAR displacement time series

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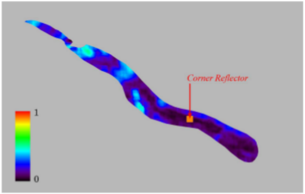
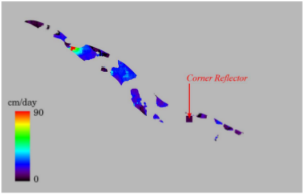
Sommaire

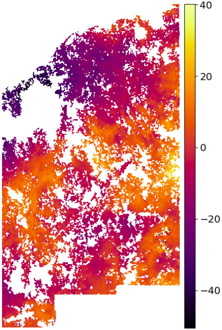
Introduction

- Missing data is a frequent issue in SAR-derived products in both space and time dimensions
- Causes : rapid surface changes, missing image, possible limitations of displacement extraction techniques.





Argentiere glacier, offset tracking of TerraSAR-X in Summer 2010 (Fallourd et al. 2011)



Interferogram over land area, Mexico (Isterre)

Motivation of the study

How can we manage the presence of **spatio-temporal** missing data in time series ?

Develop a gap-filling method capable of handling :

1. Random gaps, correlated gaps in space and time
2. Displacement signal complexity : mixed low and high frequency patterns
3. Different noise behavior : random, correlated in space and time

The EM-EOF method

- Based on empirical orthogonal functions (EOFs) : the data can be expressed in terms of EOF modes which describe a temporal or spatial variability
- Data can be interpolated by first initializing the missing values
- The EM principle is used : (1) first estimation of the data gaps with their expected value (Expectation step) and (2) minimization of the reconstruction error between reconstructed and initial data until convergence (Maximization step).

The EM-EOF method : data

- Let $X(\mathbf{s}, t)$ be a spatio-temporal field containing the values of X at position $\mathbf{s}$ and time t :

$$X = (\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n) = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{p1} & x_{p2} & x_{p3} & \cdots & x_{pn} \end{pmatrix}$$

$(x_{ij})_{1 \leq i \leq p, 1 \leq j \leq n}$ is the value at position $\mathbf{s}_i$ and time t_j and may be missing.

- Missing values are then initialized by an appropriate value (first guess).

The EM-EOF method

- The sample temporal covariance is first estimated :

$$\hat{C} = \frac{1}{p-1}(X - \mathbf{1_n}\bar{X})^T(X - \mathbf{1_n}\bar{X})$$

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- EOFs can be used to express $\hat{C}$ in terms of EOF modes :

$$\hat{C} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^T + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^T + \cdots + \lambda_n \mathbf{u}_n \mathbf{u}_n^T$$

The EM-EOF method

- X is reconstructed with M number of EOFs :

$$\boxed{X' = \sum_{i=1}^{n} a_i \mathbf{u}_i^t \rightarrow \hat{X} = \sum_{i=1}^{M \ll n} a_i \mathbf{u}_i^t}$$

with $a_i = X'\mathbf{u}_i$ are the Principal Components (PCs) of the anomaly field (X').

- The first EOF modes capture the main temporal dynamical behavior of the signal whereas other modes represent various perturbations (Prébet et al. 2019).

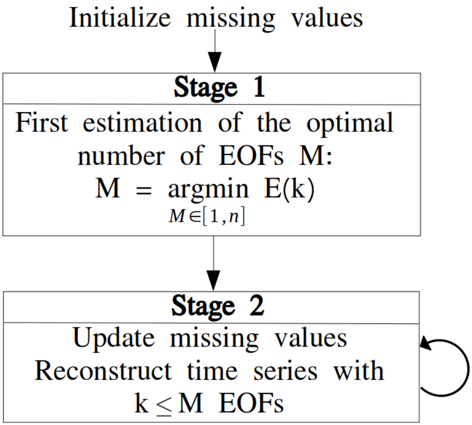
Cross-validation

- The reconstruction error is computed using a cross-validation root-mean-square error (cross-RMSE) :

$$\boxed{E(k) = \left[\frac{1}{N}\sum_{k=1}^{N} |\hat{\mathcal{X}}_k - \mathcal{X}|^2\right]^{1/2}}$$

- The optimal number of EOF modes M minimizes $E(k)$: $\underset{M\in[1,n]}{\arg\min}\, E(k)$

A 2-stage method



Numerical simulations : setup

- Displacement fields with different complexity are computed :

	$g(r,t)$	Order
g_1	$(1-0.5r)t$	1
g_2	$g_1+\sin(2\pi f_1 t)\cos(2\pi f_1 r)$	2
g_3	$g_2+0.5\cos(2\pi f_2 t)\cos(2\pi f_3 r)$	3
g_4	$g_3+0.1\sin(2\pi f_4 t)\cos(2\pi f_5 r)$	4

TABLE – $f_1=0.25$, $f_2=0.75$, $f_3=2.5$, $f_4=1.25$, $f_5=5$.

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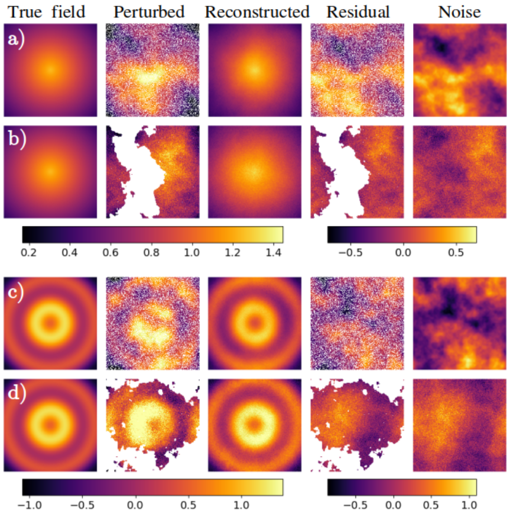
- Type of noise : random ($\mathcal{N}(0,1)$), spatially and spatio-temporally correlated
- Type of gaps : random, correlated
- SNR = [0.5,4.5]
- Gaps [0,80]%

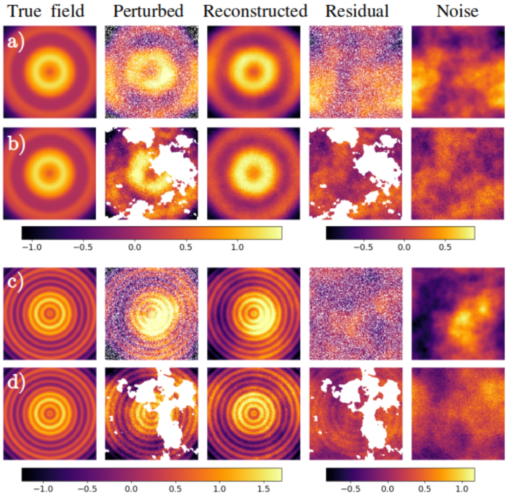
Numerical simulations : 1st and 2nd order fields



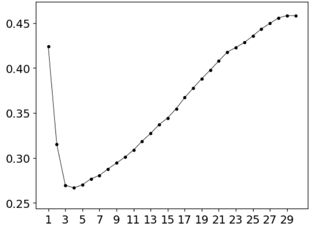
SNR and % of gaps are fixed :

- SNR = 1.5
- 30% of gaps

Numerical simulations : 3rd and 4th order fields



Number of EOF modes vs. cross-RMSE :

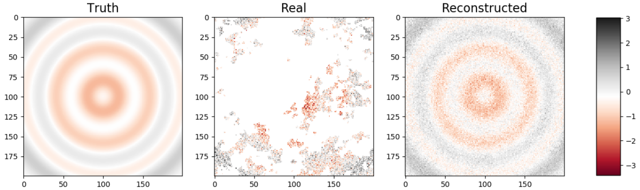


→ Minimum of the error found at the signal order

Worst case scenarii

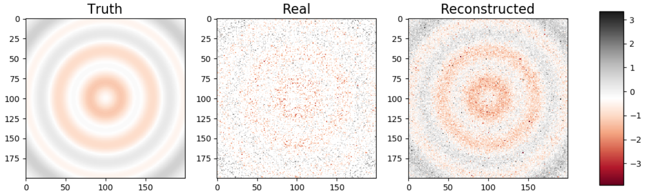
Correlated gaps :

1 EOF - 70% gaps - SNR=0.52 - 72 iterations

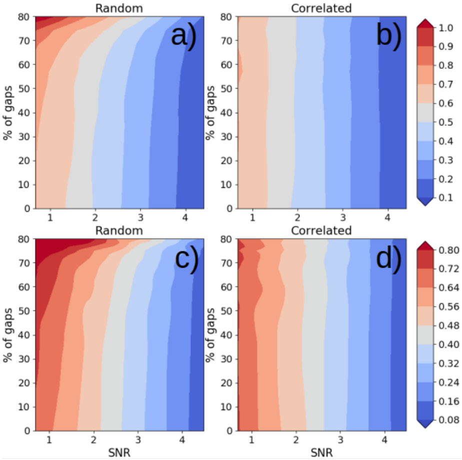


Random gaps :

1 EOF - 70% gaps - SNR=0.52 - 326 iterations



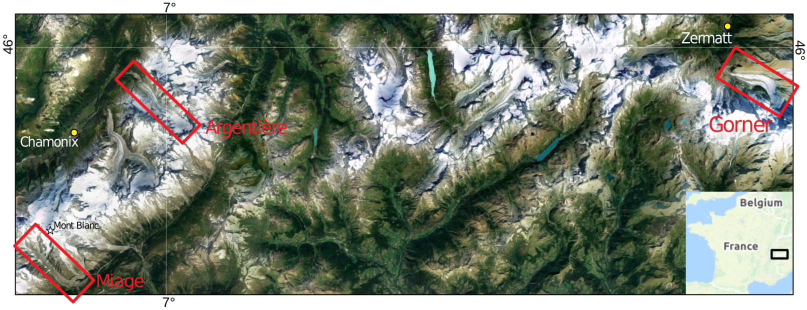
Error maps



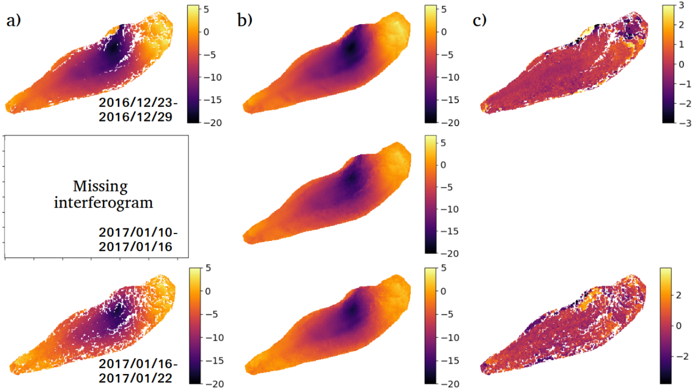
- The method is more sensitive to SNR than the % of gaps
- Random gaps affect more the reconstruction than correlated gaps
- Initialization value affects the time of convergence but not the estimation of *M*

Application to alpine glaciers

Areas of interest :



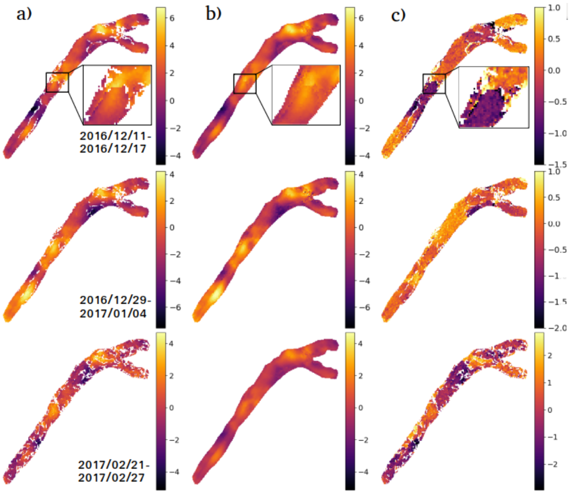
Gorner Glacier



A. Hippert-Ferrer, Y. Yan and P. Bolon, EM-EOF : gap-filling in incomplete SAR displacement time series, 2019, submitted.

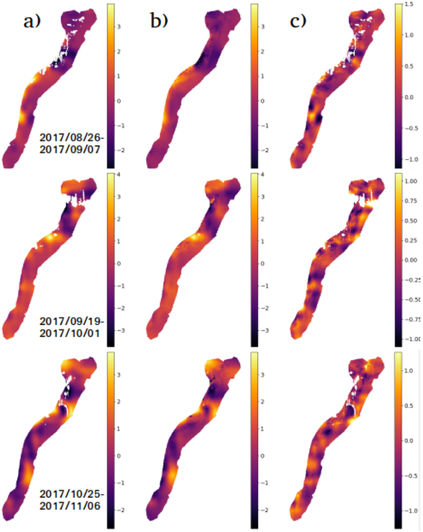
- Number of EOF modes : 3
- Consistent pattern in missing data areas
- Missing interferogram is reconstructed by adding the temporal mean to the anomaly

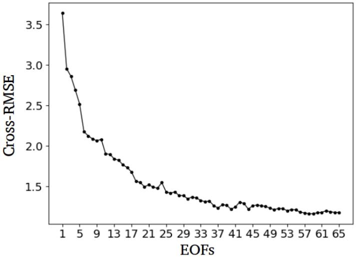
Miage Glacier



- Number of EOF modes : 2
- Discontinuities in the residuals due to phase jumps in the original interferogram
- Detection and correction of inconsistencies

Argentière Glacier





- Very low SNR and strong correlated gaps in space and time
- Strong mixing between displacement signal and noise
- Global agreement between reconstructed and initial fields

Conclusion

- EM-EOF is more sensitive to noise than gaps
- In the case of spatio-temporally correlated noise, the optimal number of EOF modes can be over-estimated
- Handling of complex cases :
  - missing interferograms
  - discontinuities due to phase jumps (due to coherence loss)
- Argentière case : limiting condition because of strong mixing between noise and displacement signal
- The method can help in increasing the effective size of a time series

Perspectives

- Use a spatio-temporal covariance matrix

  Instead of $X = (\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n)$, the spatio-temporal field is "augmented" by including a lag M :

$$\mathcal{X} = \begin{pmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_M \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_{n-M+1} & \mathbf{x}_{n-M+2} & \cdots & \mathbf{x}_n \end{pmatrix}$$

with $\mathbf{x}_t = (x_{t,1}, \dots x_{t,p})$

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- Wrapped interferogram (complex)
- We are producing augmented data : assimilation to a glacier flow model ?
- Other sources of displacement : seismic, volcanoes

Thank you. Any question ?

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[1] R. Fallourd, O. Harant, E. Trouvé and P. Bolon. "Monitoring temperate glacier displacement by multi-temporal TerraSAR-X images and continuous GPS measurements." *J-STARS*, 4 : 372-386, 2011.

[2] R. Prébet, Y. Yan, M. Jauvin and E. Trouvé. "A data-adaptive EOF based method for displacement signal retrieval from InSAR displacement measurement time series for decorrelating targets." *TGRS*, 2019 (Early access)

[3] J. M. Beckers and M. Rixen. "EOF Calculation and Data Filling from Incomplete Oceanographic Datasets". *J. Atmos. Oceanic Technol*, 20(12) : 1836-1856, 2003.